Arithmetic and Geometric Sequences

In an Arithmetic Sequence the difference between consecutive terms is constant. This difference is called the common difference.

In a geometric sequence, the ratio between consecutive terms is constant. This ratio is called the common ratio.

Ex: Is the sequence arithmetic, geometric or neither? If so, identify the common difference or common ratio.

1. 2, 5, 7, 12, ...
   3 2
   Neither
2. 48, 45, 42, 39, ...
   Arithmetic
   $d = -3$
3. 7, 10, 13, 16, ...
   Arithmetic
   $d = 3$
4. 6, -24, 96, -384, ...
   Geometric
   $r = -4$
5. 8, 20, 32, 44, ...
   Arithmetic
   $d = 12$
6. 1, -6, 36, -216, ...
   Geometric
   $r = -6$

Arithmetic Formulas:

Recursive Formula

$a_1 = \#$

$a_n = a_{n-1} + d$

Explicit (Closed) Formula

$a_n = a_1 + (n-1)d$

a_n = Specific term

n = term #

a_1 = 1st term

d = difference

r = ratio

Geometric Formulas:

Recursive Formula

$a_1 = \#$

$a_n = a_{n-1} \cdot r$

Explicit (Closed) Formula

$a_n = a_1 \cdot r^{n-1}$

7. **Find the explicit formula, the recursive formula, and the next three terms.**

   3, 9, 15, 21, ... $d = 6$ Arithmetic

   Exp: $a_n = 3 + (n-1)6$

   $a_n = 3 + 6n - 6$

   $a_n = 6n - 3$

   Rec: $a_1 = 3$, $a_n = a_{n-1} + 6$

   27, 33, 39

8. Use the recursive formula to find the 1st term and the 10th term of the sequence.

$a_6 = 2$

$a_n = a_{n-1} - 3$

$a_n = a_1 + (n-1)d$

$a_6 = a_1 + (6-1)(-3)$

$2 = a_1 + (-15)$

$+15 \qquad +15$

$\boxed{17 = a_1}$

$a_{10} = 17 + (10-1)(-3)$

$\boxed{a_{10} = -10}$

9. Find d of the arithmetic sequence for which $a_1 = 75$ and $a_{38} = 56.5$.

$a_n = a_1 + (n-1)d$

$a_{38} = 75 + (38-1)d$

$56.5 = 75 + 37d$

$-75 \quad -75$

$-18.5 = 37d$

$\boxed{-0.5 = d}$

10. Write an explicit rule and a recursive rule for the geometric sequence. Find the next three terms.

3, 6, 12, 24, ... $r = 2$

EXP: $\boxed{a_n = 3(2)^{n-1}}$

REC: $\boxed{a_1 = 3 \quad a_n = 2a_{n-1}}$

$\boxed{48, 96, 192}$

11. Find a_{12} if $a_3 = 32$ and $r = -4$.

$a_n = a_1 \cdot r^{n-1}$

$a_3 = a_1 \cdot (-4)^{3-1}$

$32 = a_1(-4)^2$

$32 = a_1(16)$

$2 = a_1$

$a_{12} = 2(-4)^{12-1}$

$\boxed{a_{12} = -8,388,608}$

12. Find the a_{19} of the geometric sequence.

11, 33, 99, 297, ... $r = 3$

$a_{19} = 11(3)^{19-1}$

$\boxed{a_{19} = 4,261,625,379}$

HW: p. 605: 1-25 odd

p. 615: 1-7 odd, 11-17 odd, 19-27 odd