13-28, 35-40, 45-46

# Basics of Vectors

**Scalar Multiplication**

A vector can be multiplied by a real number.

- Multiplying a vector by a positive number changes its size, but not its direction.
- Multiplying a vector by a negative number changes its direction and its size. (unless its multiplied by -1)

The multiplication of a scalar, *k*, and a vector, **v**, is denoted as

$$k\mathbf{v}$$

**A scalar "scales" the <u>size</u> of the vector.

**Example 1:** Given **u** = <3, 5>, find 4**u**.

$4\langle 3, 5\rangle$

$\boxed{\langle 12, 20\rangle}$

<u>**Adding Vectors:**</u> The Triangle Method

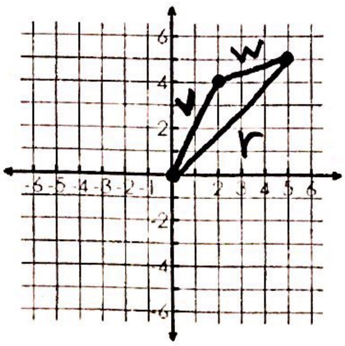
The process of (geometrically) adding two vectors is as follows:

Given vector **v** and vector **u**.

1. Draw vector **v**
2. At the terminal point of **v**, draw vector **u**
3. Draw the resultant vector (**r**) from the initial point of **v** to the terminal point of **u**.

**Example 2:** Given **v** = <2, 4> and **w** = <3, 1>

(a) Geometrically add the vectors




(b) Find the component form of the resultant

$\langle 2, 4\rangle + \langle 3, 1\rangle$

$\langle 2+3, 4+1\rangle$

$\langle 5, 5\rangle$

### Adding Vectors in Written Form

Adding vectors in written forms is fairly simple. Basically you just have to follow the order of operations.

**In component form:**

1. Multiply through by any scalars.
2. Add horizontal components, Add vertical components

**In Linear combinations:**

1. Combine like terms.

**Example 3:** Given **v** = <2, 3> and **w** = <3, -1>, find the following:

(a) **v** − **w**

$\langle 2,3\rangle - \langle 3,-1\rangle$

$\langle 2-3, 3--1\rangle$

$\boxed{\langle -1, 4\rangle}$

(b) 3**w**

$3\langle 3,-1\rangle$

$\boxed{\langle 9,-3\rangle}$

(c) 3**v** + 2**w**

$3\langle 2,3\rangle + 2\langle 3,-1\rangle$

$\langle 6,9\rangle + \langle 6,-2\rangle$

$\boxed{\langle 12, 7\rangle}$

### Unit Vectors

A unit vector is a vector of magnitude of 1 (in any direction).

To find a unit vector in a specific direction (the direction of another given vector), you must "divide" the given vector using scalar multiplication so that the new vector's magnitude is 1.

1. Find the magnitude of the given directional vector.
2. Multiply by the reciprocal of the magnitude.

**Example 4:** Find the unit vector in the same direction as <-3, 4>.

$$\|\langle -3,4\rangle\| = \sqrt{(-3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\frac{1}{5}\langle -3,4\rangle = \boxed{\left\langle -\frac{3}{5}, \frac{4}{5}\right\rangle}$$

**Example 5:** Find the vector **v** with the given magnitude and the same direction as **u**.

$\|v\| = 5$ **u** = <2, -3>

$$\|\langle 2,-3\rangle\| = \sqrt{(2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$\frac{1}{\sqrt{13}}\langle 2,-3\rangle = \left\langle \frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}}\right\rangle = \left\langle \frac{2\sqrt{13}}{13}, \frac{-3\sqrt{13}}{13}\right\rangle$$

$$5\left\langle \frac{2\sqrt{13}}{13}, \frac{-3\sqrt{13}}{13}\right\rangle = \boxed{\left\langle \frac{10\sqrt{13}}{13}, \frac{-15\sqrt{13}}{13}\right\rangle}$$