Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
DOT PRODUCTS	Recall that vector addition and subtraction are operations that result in a new vector. The **dot product** is another vector operation, however it results in a scalar.
	If $\mathbf{v} = \langle a_1, b_1 \rangle$ and $\mathbf{w} = \langle a_2, b_2 \rangle$, the dot product $\mathbf{v} \cdot \mathbf{w}$ is defined as: $v \cdot w = a_1 \cdot a_2 + b_1 \cdot b_2$
*Note: The dot product of two vectors is also called the *scalar product* or *inner product*.	**Find the dot product of the given vectors.**
	1. $\mathbf{p} = \langle 5, -2 \rangle$; $\mathbf{q} = \langle 1, -9 \rangle$ $p \cdot q = 5(1) + (-2)(-9)$ $= 5 + 18$ $= \boxed{23}$
	2. $\mathbf{r} = \langle 0, 4 \rangle$; $\mathbf{s} = \langle -2, -4 \rangle$
	3. $\mathbf{v} = 7\mathbf{i} + 8\mathbf{j}$; $\mathbf{w} = -\mathbf{i} - 9\mathbf{j}$ $v = \langle 7, 8 \rangle \quad w = \langle -1, -9 \rangle$ $v \cdot w = 7(-1) + 8(-9)$ $= \boxed{-79}$
	4. $\mathbf{c} = 3\mathbf{i} - 7\mathbf{j}$; $\mathbf{d} = -2\mathbf{i} - 2\mathbf{j}$
ORTHOGONAL Vectors	If two vectors are *perpendicular*, than their dot product is equal to 0. Vectors with a dot product of zero are also described as **orthogonal**.
	Determine whether the vectors are orthogonal.
	5. $\mathbf{m} = \langle 3, 7 \rangle$; $\mathbf{n} = \langle -28, 12 \rangle$ $m \cdot n = 3(-28) + 7(12)$ $= -84 + 84$ $= 0$ $\boxed{\text{yes}}$
	6. $\mathbf{g} = \langle -3, -2 \rangle$; $\mathbf{h} = \langle 40, -15 \rangle$ $g \cdot h = (-3)(40) + (-2)(-15)$ $= -120 + 30$ $= -90$ $\boxed{\text{NO}}$
	7. $\mathbf{x} = -5\mathbf{i} - \mathbf{j}$; $\mathbf{y} = -\mathbf{i} - 5\mathbf{j}$ $x \cdot y = -5(-1) + (-1)(-5)$ $= 5 + 5$ $= 10$ $\boxed{\text{NO}}$
	8. $\mathbf{u} = 12\mathbf{i} - 18\mathbf{j}$; $\mathbf{v} = 9\mathbf{i} + 6\mathbf{j}$
PROPERTIES of Dot Products	If $\mathbf{v}$ and $\mathbf{w}$ are vectors and k is a real number, then:
	▶ $v \cdot w = w \cdot v$
	▶ $v(w + x) = vw + vx$
	▶ $k(v \cdot w) = kv \cdot w = v \cdot kw$
	▶ $0 \cdot v = 0$
	▶ $v \cdot v = \lVert v \rVert^2$

MAGNITUDE using the Dot Product

If ______________________, then ______________________.

Use the dot product to find the magnitude of each vector.

9. $\mathbf{r} = \langle 10, 24 \rangle$

10. $\mathbf{a} = \langle -\sqrt{7}, -13 \rangle$

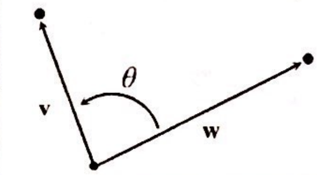
11. $\mathbf{v} = -2\mathbf{i} + 18\mathbf{j}$

12. $\mathbf{k} = 14\mathbf{i} - 3\mathbf{j}$

ANGLE BETWEEN Two Vectors

If θ is the angle between nonzero vectors $\mathbf{v}$ and $\mathbf{w}$, then:

$$\cos\theta = \frac{v \cdot w}{\|v\| \cdot \|w\|}$$



- If the angle between the vectors is 90°, then the vectors are perpendicular.
- If the angle between the vectors is 0° or 180°, then the vectors are parallel.

Find the angle between each vector pair.

13. $\mathbf{u} = \langle 8, -7 \rangle$; $\mathbf{v} = \langle -7, -3 \rangle$

$$\cos\theta = \frac{8(-7) + (-7)(-3)}{\sqrt{8^2 + (-7)^2} \cdot \sqrt{(-7)^2 + (-3)^2}}$$

$$\cos\theta = \frac{-35}{\sqrt{113} \cdot \sqrt{58}}$$

$$\cos\theta = \frac{-35}{\sqrt{6554}}$$

$$\boxed{\cos^{-1}\theta = 115.62°}$$

14. $\mathbf{m} = \langle 0, 6 \rangle$; $\mathbf{n} = \langle -3, 5 \rangle$

15. $\mathbf{a} = 6\mathbf{i} + 7\mathbf{j}$; $\mathbf{b} = 9\mathbf{i} - 6\mathbf{j}$

16. $\mathbf{p} = \mathbf{i} + 2\mathbf{j}$; $\mathbf{q} = -3\mathbf{i} - 6\mathbf{j}$

$$\cos\theta = \frac{(1)(-3) + (2)(-6)}{\sqrt{(1)^2 + (2)^2} \cdot \sqrt{(-3)^2 + (-6)^2}}$$

$$\cos\theta = \frac{-15}{\sqrt{5} \cdot \sqrt{45}}$$

$$\cos\theta = \frac{-15}{\sqrt{225}}$$

$$\boxed{\cos^{-1}\theta = 180°}$$