

Double Angle Formulas

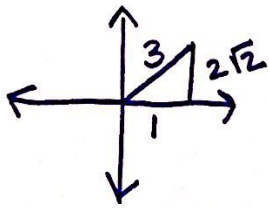
$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta \quad \sin 2\theta = 2 \sin \theta \cos \theta \quad \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

Ex: Find the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ given the value and the interval.

1. $\cos \theta = \frac{1}{3}$ ($0^\circ - 90^\circ$) QI



$$1^2 + y^2 = 3^2$$

$$1 + y^2 = 9$$

$$\sqrt{y^2} = \sqrt{8}$$

$$y = 2\sqrt{2}$$

$$\sin \theta = \frac{y}{r} = \frac{2\sqrt{2}}{3}$$

$$\tan \theta = \frac{y}{x} = 2\sqrt{2}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{2\sqrt{2}}{3} \right) \left(\frac{1}{3} \right)$$

$$= \boxed{\frac{4\sqrt{2}}{9}}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{1}{3} \right)^2 - \left(\frac{2\sqrt{2}}{3} \right)^2$$

$$= \frac{1}{9} - \frac{8}{9}$$

$$= \boxed{-\frac{7}{9}}$$

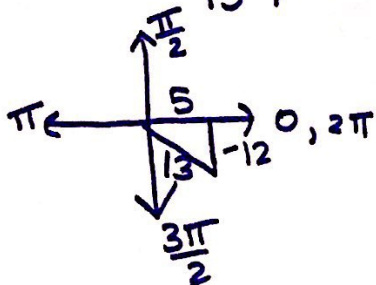
$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2(2\sqrt{2})}{1 - (2\sqrt{2})^2}$$

$$= \frac{4\sqrt{2}}{1 - 8} = \boxed{\frac{4\sqrt{2}}{-7}}$$

2. $\sec \theta = \frac{13}{5}$ ($\frac{3\pi}{2}, 2\pi$) QIV

$$\cos \theta = \frac{5}{13}$$



$$\sin \theta = \frac{y}{r} = -\frac{12}{13}$$

$$\tan \theta = \frac{y}{x} = -\frac{12}{5}$$

$$\sin 2\theta = \frac{2(-\frac{12}{13})(\frac{5}{13})}{1 - (-\frac{12}{13})^2} = \frac{-120}{169}$$

$$\cos 2\theta = \left(\frac{5}{13} \right)^2 - \left(-\frac{12}{13} \right)^2$$

$$= \frac{25}{169} - \frac{144}{169}$$

$$= \boxed{-\frac{119}{169}}$$

$$\tan 2\theta = \frac{2\left(-\frac{12}{5}\right)}{1 - \left(-\frac{12}{5}\right)^2}$$

$$= \frac{-\frac{24}{5}}{1 - \frac{144}{25}}$$

$$= \frac{-\frac{24}{5}}{-\frac{119}{25}}$$

$$= -\frac{24}{5} \cdot \frac{25}{-119}$$

$$= \boxed{\frac{120}{119}}$$

Hw: p. 352: 1-8