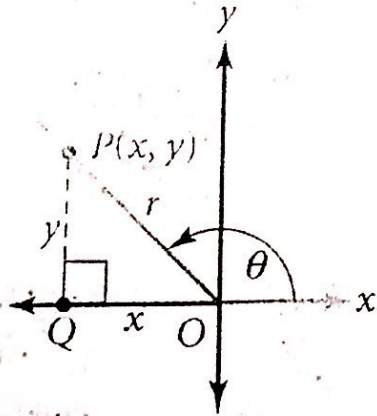


Finding exact values of Trig Functions

Let $P(x, y)$ be a point on the terminal side of θ in standard position.

The distance from the origin to P is given by $r = \sqrt{x^2 + y^2}$.



$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

Cosecant

$$\sec \theta = \frac{r}{x}$$

Secant

$$\cot \theta = \frac{x}{y}$$

Cotangent

Ex: The given point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

1. $(7, 24)$

$$7^2 + 24^2 = r^2$$

$$\sqrt{625} = \sqrt{r^2}$$

$$25 = r$$

$$x = 7$$

$$y = 24$$

2. $(-2, 9)$

$$(-2)^2 + 9^2 = r^2$$

$$\sqrt{85} = \sqrt{r^2}$$

$$r = \sqrt{85}$$

$$x = -2$$

$$y = 9$$

$$\sin \theta = \frac{y}{r} = \frac{24}{25}$$

$$\csc \theta = \frac{25}{24}$$

$$\sin \theta = \frac{9 \cdot \sqrt{85}}{\sqrt{85} \cdot \sqrt{85}} = \frac{9\sqrt{85}}{85}$$

$$\csc \theta = \frac{\sqrt{85}}{9}$$

$$\cos \theta = \frac{x}{r} = \frac{7}{25}$$

$$\sec \theta = \frac{25}{7}$$

$$\cos \theta = \frac{-2 \cdot \sqrt{85}}{\sqrt{85} \cdot \sqrt{85}} = \frac{-2\sqrt{85}}{85}$$

$$\sec \theta = \frac{\sqrt{85}}{-2}$$

$$\tan \theta = \frac{y}{x} = \frac{24}{7}$$

$$\cot \theta = \frac{7}{24}$$

$$\tan \theta = \frac{9}{-2}$$

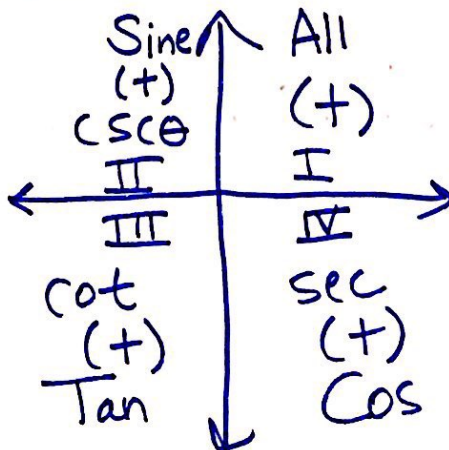
$$\cot \theta = \frac{-2}{9}$$

Ex: State the quadrant in which θ lies.

3. $\sin \theta < 0$ and $\tan \theta < 0$

(-) (-)

IV



4. $\cot \theta > 0$ and $\tan \theta > 0$

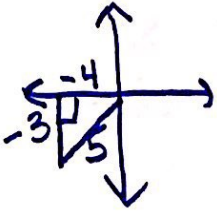
(+) (+)

I, III

Ex: Find the values of the six trigonometric functions of θ .

5. $\cos \theta = -\frac{4}{5}$ where θ lies in Quadrant III

$$\begin{aligned} x &= -4 \\ r &= 5 \\ y &= -3 \end{aligned}$$



$$\begin{aligned} (-4)^2 + y^2 &= (5)^2 \\ 16 + y^2 &= 25 \\ \sqrt{y^2} &= \sqrt{9} \\ y &= -3 \end{aligned}$$

$$\sin \theta = \frac{-3}{5}$$

$$\tan \theta = \frac{-3}{-4} = \frac{3}{4}$$

$$\csc \theta = \frac{-5}{3}$$

$$\sec \theta = \frac{-5}{4}$$

$$\cot \theta = \frac{4}{3}$$

Ex: Find the indicated trigonometric value.

Function

7. $\cot \theta = -3 = \frac{x}{y}$

$x = -3$

$y = 1$

$r = \sqrt{10}$

$$\begin{aligned} (-3)^2 + (1)^2 &= r^2 \\ \sqrt{10} &= \sqrt{r^2} \end{aligned}$$

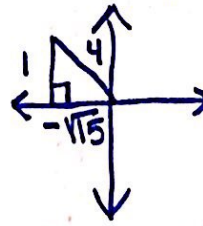
$$\sin \theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

Quadrant

II

6. $\csc \theta = 4$ where $\cot \theta < 0$

$$\csc \theta = \frac{r}{y}$$



$$\begin{aligned} x &= -\sqrt{15} \\ r &= 4 \\ y &= 1 \end{aligned}$$

$$x^2 + (1)^2 = (4)^2$$

$$\begin{aligned} x^2 + 1 &= 16 \\ \sqrt{x^2} &= \sqrt{15} \\ x &= -\sqrt{15} \end{aligned}$$

$$\sin \theta = \frac{1}{4}$$

$$\sec \theta = \frac{-4 \cdot \sqrt{15}}{\sqrt{15} \cdot \sqrt{15}} = \frac{-4\sqrt{15}}{15}$$

$$\cos \theta = \frac{-\sqrt{15}}{4}$$

$$\tan \theta = \frac{1 \cdot \sqrt{15}}{-\sqrt{15} \cdot \sqrt{15}} = \frac{-\sqrt{15}}{15}$$

Trigonometric Value

$\sin \theta$

$$\cot \theta = -\sqrt{15}$$

8. $\csc \theta = -2$

IV

$\cot \theta$

p. 251 : 1-7 odd, 33-39 odd