29-30, 41-44, 51-54, 61-72

Forms of Vectors and Operations on Vectors

Linear Combinations

The linear combination form of a vector uses scalars of the standard unit vectors

$\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$

to write the position vector.

To write any given vector as a linear combination, write the vector as a sum of its components in terms of standard unit vectors.

Example 1: Write vector $\langle 3, 5 \rangle$ as a linear combination of **i** and **j**

$\boxed{3i + 5j} \Rightarrow 3\langle 1, 0\rangle + 5\langle 0, 1\rangle$

$\langle 3, 0\rangle + \langle 0, 5\rangle$

$\langle 3, 5\rangle$

Example 2: Given $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{u} = 5\mathbf{i} - 2\mathbf{j}$

a) $\mathbf{v} + \mathbf{u}$

$2i + 3j + 5i - 2j$

$7i + 1j$

$\boxed{\langle 7, 1\rangle}$

b) $3\mathbf{u}$

$3(5i - 2j)$

$15i - 6j$

$\boxed{\langle 15, -6\rangle}$

c) $2\mathbf{u} - 3\mathbf{v}$

$2(5i - 2j) - 3(2i + 3j)$

$10i - 4j - 6i - 9j$

$4i - 13j$

$\boxed{\langle 4, -13\rangle}$

Example 3: Find the unit vector in the direction of the given vector

a) $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ $\langle 2, 3\rangle$

b) $\mathbf{u} = -4\mathbf{j}$ $\langle 0, -4\rangle$

$\|\vec{v}\| = \sqrt{(2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$

$\frac{1}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}}{13} \langle 2, 3\rangle$

$\boxed{\left\langle \frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right\rangle}$

Example 4: Write the vector as a linear combination of the standard unit vector **i** and **j**.

Initial Point: (-4, 3) Terminal Point: (5, -2)

$\langle 5 - -4, -2 - 3\rangle$

$\langle 9, -5\rangle \leftarrow$ component form

$\boxed{9i - 5j} \leftarrow$ linear combination form

Direction Angles of Vectors

The direction angle of a vector is the Standard angle of the vector when it is in standard position.

Using its horizontal (x) and vertical (y) components, we can find the direction angle, θ, of a vector using:

$$tan\theta = \frac{y}{x} \qquad \tan^{-1}\left(\frac{y}{x}\right)$$

Trigonometric Forms of Vectors

Considering that the horizontal (x) component of any right triangle is related to the cosine and the vertical (y) component is related to the Sine, we can put any vector into it's Trigonometric (component) Form based on its magnitude and direction angle.

Given a vector, **v** = <x, y> with direction angle θ, the trigonometric form of **v** is

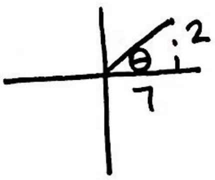
$$\vec{v} = \|\vec{v}\| \langle \cos\theta, \sin\theta \rangle$$
$$= \langle \|\vec{v}\|\cos\theta, \|\vec{v}\|\sin\theta \rangle$$

****Note:** The direction angle is the angle, θ, that the vector in standard position makes with the positive x-axis where $0 \le \theta < 2\pi$. or 0° & 360°

Example 5: Find the magnitude and direction angle of the vector **v**.

a) **v** = <7, 2>

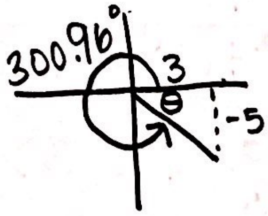
$$\|\vec{v}\| = \sqrt{(7)^2+(2)^2} = \boxed{\sqrt{53}}$$



$$\tan^{-1}\left(\frac{2}{7}\right) = \boxed{15.95°}$$

b) **v** = 3**i** – 5**j** $\langle 3, -5 \rangle$

$$\|\vec{v}\| = \sqrt{(3)^2+(-5)^2} = \boxed{\sqrt{34}}$$



$$\tan^{-1}\left(-\frac{5}{3}\right) = -59.04$$

$$-59.04 + 360 = \boxed{300.96°}$$

Example 6: Find the component form of **v** given its magnitude and the angle it makes with the positive x-axis. Sketch **v**.

Magnitude: $\|v\| = 27$ Angle: $\theta = 140°$

$$27\langle \cos 140, \sin 140 \rangle$$
$$\langle 27\cos 140, 27\sin 140 \rangle$$
CF: $\boxed{\langle -20.68, 17.36 \rangle}$