Geometric Series

Geometric Series: the expression for the sums of the terms of a geometric sequence.

Converges: When $|r|<1$. When you get closer & closer to the sum, approaching a specific #.

Diverges: When $|r| \geq 1$. Approaching no limit, no specific #.

Sum of an Infinite Geometric Series:

$$S = \frac{a_1}{1-r}$$

(you will have a sum when $|r|<1$)

Determine whether the sequence is *convergent* or *divergent*.

1. $a_n = \frac{64}{2n}$

Approaching Zero

Convergent

2. $a_n = 3(-1)^n$

Divergent

No approaching a specific #

Ex: Evaluate the infinite geometric series.

3. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

$r = \frac{1}{8} \div \frac{1}{4} = \frac{1}{2}$

$\frac{1}{2} < 1$ ✓

$S = \frac{a_1}{1-r}$

$S = \frac{1}{1-\frac{1}{2}} = \boxed{2}$

4. $0.25+(-1.25)+6.25+\ldots$

$r = 6.25 \div (-1.25) = -5$

$|-5|$

$5 \not< 1$

DNE

5. $\sum_{n=1}^{\infty} 120(0.8)^{n-1}$

a_1, r

$0.8 < 1$ ✓

$S = \frac{120}{1-0.8} = \boxed{600}$

Geometric Means: Find r: $a_n = a_1 r^{n-1}$

6. Write a sequence that has 2 geometric means between 480 and -7.5.

480, —, —, -7.5

* Find r first

$a_n = a_1 r^{n-1}$

$-7.5 = 480r^{4-1}$

$-7.5 = 480r^3$

$\sqrt[3]{-0.015625} = \sqrt[3]{r^3}$

$r = -0.25$

480, -120, 30, -7.5

Sum of a Finite Geometric Series:

$$S_n = a_1\left(\frac{1-(r)^n}{1-r}\right) \qquad S_n = \frac{a_1 - a_n r}{1-r}$$

7. Find the sum of the first 6 terms of the sequence 8 + 14 + 24.5 + ... $n=6, a_1=8, r=1.75$

$$S_6 = 8\left(\frac{1-(1.75)^6}{1-1.75}\right)$$

$S_6 = 295.71$

8. Find the sum of the first n terms of the geometric series. $a_1 = 3; a_n = 768; r = -2$

$$S_n = \frac{3 - 768(-2)}{1-(-2)}$$

$S_n = 513$

P. 595: 19-27 odd, 33-37 odd
P. 615-616: 41-63 odd

9. Find the sum: $\sum_{n=2}^{7} 3(5)^{n-1}$

* Find a_1 first

$a_1 = 3(5)^{2-1}$

$a_1 = 3(5)^1$

$a_1 = 15$, $r = 5$, $n = 6$

$$S_6 = 15\left(\frac{1-(5)^6}{1-5}\right)$$

$S_6 = 58,590$