

Inverse Composition of Trigonometric Expressions

Evaluate each inverse expression for principal values only and write your final answer as an exact value. If no solution exists put "DNE".

1. ~~$\sin(\sin^{-1} \frac{3}{4})$~~

$$\boxed{\frac{3}{4}}$$

2. $\sin^{-1}(\sin \frac{\pi}{2})$

$$\sin^{-1}(1)$$

$$\boxed{\frac{\pi}{2}}$$

3. ~~$\cos(\cos^{-1} \frac{2}{9})$~~

$$\boxed{\frac{2}{9}}$$

4. $\cos^{-1}(\cos \pi)$

$$\cos^{-1}(-1)$$

$$\boxed{\pi}$$

5. $\tan(\tan^{-1} \frac{\pi}{4})$

$$\tan(1)$$

$$\boxed{\frac{\pi}{4}}$$

6. $\tan^{-1}(\tan \frac{\pi}{3})$

$$\boxed{\frac{\pi}{3}}$$

7. $\cos(\tan^{-1} 0)$

$$\cos(0\pi)$$

$$\boxed{1}$$

$\cos(0\pi)$

$$\boxed{1}$$

8. $\sin^{-1}(\cos \frac{\pi}{2})$

$$\sin^{-1}(0)$$

$$\boxed{0}$$

9. ~~$\sin(\cos^{-1}(\frac{\sqrt{2}}{2}))$~~

$$\sin(\frac{\pi}{4})$$

$$\boxed{\frac{\sqrt{2}}{2}}$$

10. $\arctan(\sin \frac{\pi}{2})$

$$\arctan(1)$$

$$\boxed{\frac{\pi}{4}}$$

11. $\arcsin(\sin \frac{7\pi}{4})$

$$\arcsin(-\frac{\sqrt{2}}{2})$$

$$\boxed{-\frac{\pi}{4}}$$

12. ~~$\sin(2\cos^{-1} \frac{\sqrt{2}}{2})$~~

$$\sin(2(\frac{\pi}{4}))$$

$$\sin(\frac{\pi}{2})$$

$$\boxed{1}$$

13. $\sin(\tan^{-1}(1) - \sin^{-1}(1))$

$$\sin(\frac{\pi}{4} - \frac{\pi}{2})$$

$$\sin(\frac{\pi}{4} - \frac{2\pi}{4})$$

$$\sin(-\frac{\pi}{4})$$

$$\boxed{-\frac{\sqrt{2}}{2}}$$

14. $\cos(\tan^{-1}(1) - \sin^{-1}(1))$

$$\cos(\frac{\pi}{4} - \frac{2\pi}{4})$$

$$\cos(-\frac{\pi}{4})$$

$$\boxed{\frac{\sqrt{2}}{2}}$$

15. $\cos(\cos^{-1}(0) + \sin^{-1} \frac{1}{2})$

$$\cos(\frac{\pi}{2} + \frac{\pi}{6})$$

$$\cos(\frac{3\pi}{6} + \frac{\pi}{6})$$

$$\cos(\frac{4\pi}{6})$$

$$\cos(\frac{2\pi}{3})$$

$$\boxed{-\frac{1}{2}}$$

Inverse Composition of Trigonometric Expressions

Values Not on the Unit Circle: Draw a triangle $\cos = \frac{x}{r}$ $\sin = \frac{y}{r}$ $\tan = \frac{y}{x}$

16. $\sin\left(\arctan\frac{5}{12}\right) \frac{y}{x}$

$5^2 + 12^2 = r^2$
 $\sqrt{169} = \sqrt{r^2}$
 $13 = r$

$\sin = \frac{y}{r}$
 $\frac{5}{13}$

17. $\cos\left(\tan^{-1}\left(-\frac{3}{4}\right)\right) \frac{y}{x}$

$4^2 + (-3)^2 = r^2$
 $\sqrt{25} = \sqrt{r^2}$
 $5 = r$

$\cos = \frac{x}{r}$
 $\frac{4}{5}$

18. $\tan\left(\arccos\frac{5}{4}\right) \frac{x}{r}$

$5^2 + y^2 = 4^2$
 $25 + y^2 = 16$
 $y^2 = -9$

$\frac{5}{4} > 1$
DNE

19. $\sin\left(\arccos\left(\frac{1}{x}\right)\right) \frac{x}{r}$

$1^2 + y^2 = x^2$
 $1 + y^2 = x^2$
 $\sqrt{y^2} = \sqrt{x^2 - 1}$

$\sin = \frac{y}{r}$
 $\frac{\sqrt{x^2 - 1}}{x}$
 $y = \sqrt{x^2 - 1}$

20. $\cos\left(\arcsin\frac{x}{1}\right) \frac{y}{r}$

$\cos = \frac{x}{r}$
 $\frac{\sqrt{1 - x^2}}{1} = \sqrt{1 - x^2}$

17. $\cos\left(\sin^{-1}\left(\frac{-12}{13}\right)\right) \frac{y}{r}$

$\cos = \frac{x}{r}$
 $\frac{5}{13}$

$x^2 + a^2 = 12$
 $x^2 + a^2 = 1$
 $\sqrt{a^2} = \sqrt{1 - x^2}$
 $a = \sqrt{1 - x^2}$