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Multiplicative Inverse of a 2 x 2 Matrix	
Why do we need them?	How do we find them?
When working with equations, you may need to cancel out a matrix in order to find a solution. You can multiply a matrix by its inverse to cancel it out. Notation: A^{-1}	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ det ≠ 0 have an inverse.

If det = 0, the matrix has No inverse. (A matrix with no inverse is called a singular matrix. **Inverse matrices answer questions about the existence of solutions for systems of equations.

Multiplicative Identity Matrix: If you multiply a matrix by its inverse, you get the identity matrix.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, etc. 1's on main diagonal & 0's everywhere else.

(Identity Matrix)

1. Verify Inverses: $\begin{bmatrix} 2 & 1 \\ 2.5 & 1 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 2(-2)+1(5) & 2(2)+1(-4) \\ 2.5(-2)+1(5) & 2.5(2)+1(-4) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Identity Matrix $\boxed{\text{Yes, they inverses}}$

2. Find the inverse: $B = \begin{bmatrix} 2 & -4 \\ 1 & -3 \end{bmatrix}$

$$B^{-1} = \frac{1}{2(-3)-1(-4)}\begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} & -2 \\ \frac{1}{2} & -1 \end{bmatrix}$$

3. Solve the matrix equation. Find A^{-1}. $Ax=B$; put in the form $x=A^{-1}B$.

$$\begin{bmatrix} 3 & -4 \\ 4 & -5 \end{bmatrix} x = \begin{bmatrix} 0 & -22 \\ 0 & -28 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-15+16}\begin{bmatrix} -5 & 4 \\ -4 & 3 \end{bmatrix} = \frac{1}{1}\begin{bmatrix} -5 & 4 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ -4 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} -5 & 4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 0 & -22 \\ 0 & -28 \end{bmatrix}$$

$$X = \begin{bmatrix} -5(0)+4(0) & -5(-22)+4(-28) \\ -4(0)+3(0) & -4(-22)+3(-28) \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 0 & 4 \end{bmatrix}$$

4. Solve the systems of equations using matrices:

$2x - 3y = 17$
$3x + y = 9$

$$\begin{bmatrix} 2 & -3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ 9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2--9}\begin{bmatrix} 1 & 3 \\ -3 & 2 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 1 & 3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{11} & \frac{3}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{11} & \frac{3}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}\begin{bmatrix} 17 \\ 9 \end{bmatrix}$$

2x2 2x1

2x1

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{11}(17) + \frac{3}{11}(9) \\ -\frac{3}{11}(17) + \frac{2}{11}(9) \end{bmatrix} = \begin{bmatrix} \frac{17}{11} + \frac{27}{11} \\ -\frac{51}{11} + \frac{18}{11} \end{bmatrix} = \begin{bmatrix} \frac{44}{11} \\ \frac{-33}{11} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$