

Solving Systems of Equations Using Matrices

System of Equations

$$\begin{aligned} 2x - 5y &= 23 \\ -x + 3y &= 14 \end{aligned}$$

Matrix Equation

$$\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 23 \\ 14 \end{bmatrix}$$

Ex: Convert the system of equations to a matrix equation. Identify the coefficient matrix, variable matrix and the constant matrix. Then solve and check!

1. $7x + 5y = 3$
 $3x - 2y = 22$

2. $5a + 3b = 7$
 $3a + 2b = 5$

$$\begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left[\begin{array}{cc|c} 7 & 5 & 3 \\ 3 & -2 & 22 \end{array} \right] \quad 2 \times 3$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix} \quad \boxed{(4, -5)}$$

3. $x + y + z = 2$
 $2x + y = 5$
 $x + 3y - 3z = 14$

4. $7x + 3y + 2z = 13$
 $-2x + y - 8z = 26$
 $x - 4y + 10z = -13$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 5 \\ 1 & 3 & -3 & 14 \end{array} \right]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.25 \\ 2.5 \\ -1.75 \end{bmatrix}$$

p. 392: 1-16,
19-20