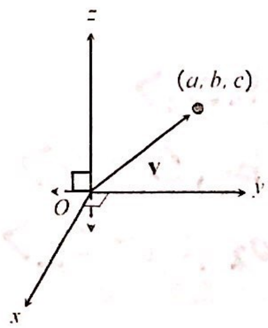
VECTORS
in Space



- In space, a vector **v** in standard position with terminal point (a, b, c) is denoted in component form as $v = \langle a, b, c \rangle$ and has a magnitude of $\|v\| = \sqrt{a^2+b^2+c^2}$.
- Using standard unit vectors $i = \langle 1, 0, 0 \rangle$, $j = \langle 0, 1, 0 \rangle$ and $k = \langle 0, 0, 1 \rangle$, the vector can also be written as the linear combination $ai + bj + ck$.
- Given any vector with initial point (x_1, y_1, z_1) and terminal point (x_2, y_2, z_2), you can write the vector in component form as $v = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$.

Find the magnitude of each vector.

4. $\mathbf{v} = \langle 4, -2, -6 \rangle$

5. $\mathbf{a} = \langle -1, 11, 5 \rangle$

6. $\mathbf{p} = 2\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$

7. $\mathbf{t} = -7\mathbf{i} - \mathbf{j} - 3\sqrt{2}\mathbf{k}$ $\quad \langle -7, -1, -3\sqrt{2} \rangle$

$$\|t\| = \sqrt{(-7)^2 + (-1)^2 + (-3\sqrt{2})^2}$$
$$= \sqrt{68} = \boxed{2\sqrt{17}}$$

2 34
2 17

Write $\overrightarrow{AB}$ in component form with the given initial and terminal points.

8. $A(-1, -4, 6)$ and $B(11, 0, -1)$

$$\overrightarrow{AB} = \langle 11 - {}^-1, 0 - {}^-4, -1 - 6 \rangle$$
$$\boxed{\langle 12, 4, -7 \rangle}$$

9. $A(13, -7, 1)$ and $B(-3, 5, -9)$

Write $\overrightarrow{AB}$ as a linear combination with the given initial and terminal points.

10. $A(7, 2, -1)$ and $B(0, 6, -3)$

11. $A(-8, -5, 2)$ and $B(-17, -5, 0)$

$$\overrightarrow{AB} = \langle -17 - {}^-8, -5 - {}^-5, 0 - 2 \rangle$$
$$= \langle -9, 0, -2 \rangle$$
$$\boxed{-9i - 2k}$$

Name:	Date:
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples
OPERATIONS OF VECTORS in Space	If $\mathbf{v} = <a_1, b_1, c_1>$ and $\mathbf{w} = <a_2, b_2, c_2>$ are vectors and k is a real number, then:

SCALAR MULTIPLCATION	$k\mathbf{v} = \langle ka_1, kb_1, kc_1 \rangle$
ADDITION	$\mathbf{v} + \mathbf{w} = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$
SUBTRACTION	$\mathbf{v} - \mathbf{w} = \langle a_1 - a_2, b_1 - b_2, c_1 - c_2 \rangle$

Directions: Find each of the following for $\mathbf{a} = \langle -1, -5, -3 \rangle$, $\mathbf{b} = \langle 2, 7, -10 \rangle$, and $\mathbf{c} = \langle 3, -4, 9 \rangle$.

1. $5\mathbf{a} - 2\mathbf{b} + \mathbf{c}$

$5\langle -1, -5, -3 \rangle - 2\langle 2, 7, -10 \rangle + \langle 3, -4, 9 \rangle$

$\langle -5, -25, -15 \rangle - \langle 4, 14, -20 \rangle + \langle 3, -4, 9 \rangle$

$\langle -5 - 4 + 3, -25 - 14 + -4, -15 - -20 + 9 \rangle$

$\boxed{\langle -6, -43, 14 \rangle}$

2. $-3\mathbf{a} + 7\mathbf{b} - 4\mathbf{c}$

Directions: Find each of the following for $\mathbf{x} = -7\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, $\mathbf{y} = 2\mathbf{i} - \mathbf{j} - 6\mathbf{k}$, and $\mathbf{z} = -3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

3. $4\mathbf{x} + 3\mathbf{y}$

4. $-\mathbf{x} - 5\mathbf{y} + 8\mathbf{z}$

DOT PRODUCT	The **dot product** of two products in space is similar to finding the dot product of two vectors in a plane. If $\mathbf{v} = <a_1, b_1, c_1>$ and $\mathbf{w} = <a_2, b_2, c_2>$, the dot product $\mathbf{v} \cdot \mathbf{w}$ is defined as: $v \cdot w = a_1 \cdot a_2 + b_1 \cdot b_2 + c_1 \cdot c_2$ The vectors are orthogonal if and only if $v \cdot w = 0$.

Directions: Find the dot product of the given vectors, then determine if they are orthogonal.

5. $\mathbf{u} = \langle -9, -2, 12 \rangle$; $\mathbf{v} = \langle -2, 3, -1 \rangle$

6. $\mathbf{p} = \langle 4, -7, -10 \rangle$; $\mathbf{q} = \langle 0, -5, 3 \rangle$

7. $\mathbf{r} = -2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$; $\mathbf{s} = -9\mathbf{i} + 4\mathbf{k}$

8. $\mathbf{a} = 17\mathbf{i} - 8\mathbf{j} - 2\mathbf{k}$; $\mathbf{b} = -4\mathbf{i} - 10\mathbf{j} + 6\mathbf{k}$

$a \cdot b = 17(-4) + (-8)(-10) + (-2)(6)$

$= -68 + 80 + -12$

$= \boxed{0}$

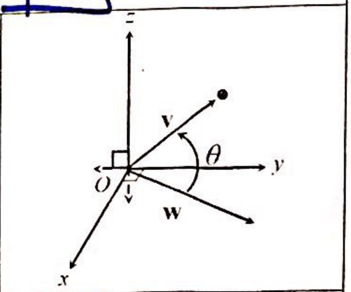
$\boxed{\text{Yes}}$

ANGLE BETWEEN

Two Vectors

As with vectors in the plane, if θ is the angle between nonzero vectors $\mathbf{v}$ and $\mathbf{w}$, then:

$$\cos\theta = \frac{v \cdot w}{\|v\| \cdot \|w\|}$$



Directions: Find the angle between each vector pair to the nearest tenth of a degree.

9. $\mathbf{u} = \langle 4, -2, -1 \rangle$; $\mathbf{v} = \langle -5, 3, -8 \rangle$

10. $\mathbf{m} = \langle -8, 1, 10 \rangle$; $\mathbf{n} = \langle 4, 7, -2 \rangle$

11. $\mathbf{c} = -6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$; $\mathbf{d} = -3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

$$\cos\theta = \frac{-6(-3) + (-3)(1) + 2(-2)}{\sqrt{(-6)^2 + (-3)^2 + 2^2} \cdot \sqrt{(-3)^2 + (1)^2 + (-2)^2}}$$

$$\cos\theta = \frac{11}{\sqrt{49} \cdot \sqrt{14}}$$

$$\cos\theta = \frac{11}{7\sqrt{14}}$$

$$\cos^{-1}\left(\frac{11}{7\sqrt{14}}\right)$$

$$\boxed{\theta = 65.2^\circ}$$

12. $\mathbf{v} = 8\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$; $\mathbf{w} = 12\mathbf{i} - 5\mathbf{j} - 9\mathbf{k}$

P. 514: 26-46 even

P. 522: 1-15 odd

(26) A (-4, 0, -3)   B (-4, -8, 9)

$\overrightarrow{AB} = \langle -4 - -4, -8 - 0, 9 - -3 \rangle$

$= \boxed{\langle 0, -8, 12 \rangle}$

$||AB|| = \sqrt{0^2 + (-8)^2 + (12)^2} = \sqrt{208} = \boxed{4\sqrt{13}}$

$208 \to 2 \cdot \cancel{104}$, $104 \to 2 \cdot \cancel{52}$, $52 \to 2 \cdot \cancel{26}$, $26 \to 2 \cdot 13$

unit vector: $\dfrac{v}{||v||}$

$$\frac{\langle 0, -8, 12 \rangle}{4\sqrt{13}} = \left\langle \frac{0}{4\sqrt{13}}, \frac{-8}{4\sqrt{13}}, \frac{12}{4\sqrt{13}} \right\rangle$$

$$\left\langle 0, \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$$

$$\boxed{\left\langle 0, \frac{-2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right\rangle}$$