

Simplify each expression.

$\sec^2 \theta (1 - \cos^2 \theta)$

$$\frac{\sec^2 \theta (\sin^2 \theta)}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} (\sin^2 \theta)$$

$$\frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\boxed{\tan^2 \theta}$$

2. $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} \cdot (1 + \cos \theta)$

$$\frac{\sin^2 \theta}{\sin \theta (1 + \cos \theta)} + \frac{1 + 2\cos \theta + \cos^2 \theta}{\sin \theta (1 + \cos \theta)}$$

$$\frac{\sin^2 \theta + \cos^2 \theta + 1 + 2\cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$\frac{1 + 1 + 2\cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$\frac{2 + 2\cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$\frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)}$$

$$\frac{2}{\sin \theta} = \boxed{2\csc \theta}$$

3. $\cos \theta (\sec \theta - \cos \theta)$

$$\cos \theta \left(\frac{1}{\cos \theta} - \cos \theta \right)$$

$$1 - \cos^2 \theta$$

$$\boxed{\sin^2 \theta}$$

$\frac{\sin^2 \theta + \tan^2 \theta + \cos^2 \theta}{\sec \theta}$

$$\frac{\sin^2 \theta + \cos^2 \theta + \tan^2 \theta}{\sec \theta}$$

$$\frac{1 + \tan^2 \theta}{\sec \theta}$$

$$\frac{\sec^2 \theta}{\sec \theta}$$

$$\boxed{\sec \theta}$$

5. $\frac{\sec(-x)}{\tan(-x)}$

$$\frac{\sec x}{-\tan x}$$

$$\frac{1}{\cos x} \cdot \frac{1}{-\frac{\sin x}{\cos x}}$$

$$\frac{1}{-\sin x}$$

$$\boxed{-\csc x}$$

6. $\frac{\csc^2 \theta - 1}{\csc^2 \theta} + \frac{\sec^2 \theta - 1}{\sec^2 \theta}$

$$\frac{\cot^2 \theta}{\csc^2 \theta} + \frac{\tan^2 \theta}{\sec^2 \theta}$$

$$\frac{\frac{\cos^2 \theta}{\sin^2 \theta}}{\frac{1}{\sin^2 \theta}} + \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta}}$$

$$\cos^2 \theta + \sin^2 \theta$$

$$\boxed{1}$$

II. Solve each equation on the interval $[0, 2\pi)$.

7. $4\sin x + 1 = 2\sin x$
 $\frac{-2\sin x}{-2\sin x}$

$$2\sin x + 1 = 0$$

$$2\sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

8. $\tan^2 \theta - 3 = 0$

$$\sqrt{\tan^2 \theta} = \sqrt{3}$$

$$\tan \theta = \pm \sqrt{3}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

9. $3\sec^2 \theta - 4 = 0$

$$3\sec^2 \theta = 4$$

$$\sec^2 \theta = \frac{4}{3}$$

$$\sqrt{\cos^2 \theta} = \sqrt{\frac{3}{4}}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

10. $6\cos^2 \theta - 6 = 0$

$$6\cos^2 \theta = 6$$

$$\sqrt{\cos^2 \theta} = \sqrt{1}$$

$$\cos \theta = \pm 1$$

$$\theta = 0\pi, \pi$$

11. $3\sin^2 \theta + 2\sin \theta = 1$

$$3\sin^2 \theta + 2\sin \theta - 1 = 0$$

$$\frac{-2 \pm \sqrt{4 - 4(3)(-1)}}{2(3)} (3\sin^2 \theta + 3\sin \theta) - (\sin \theta - 1) = 0$$

$$3\sin \theta (\sin \theta + 1) - 1(\sin \theta + 1) = 0$$

$$(\sin \theta + 1)(3\sin \theta - 1) = 0$$

$$\sin \theta + 1 = 0 \quad 3\sin \theta - 1 = 0$$

$$\sin \theta = -1 \quad 3\sin \theta = 1$$

$$\sin \theta = \frac{1}{3}$$

$$\theta = \frac{3\pi}{2}$$

14. $\sin^2 x - 3\cos^2 x = 0$

$$1 - \cos^2 x - 3\cos^2 x = 0$$

$$-4\cos^2 x + 1 = 0$$

$$-4\cos^2 x = -1$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{4}}$$

$$\cos x = \pm \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

15. $\sqrt{3}\tan 3x = 1$

$$\tan 3x = \frac{1}{\sqrt{3}}$$

$$\tan 3x = \frac{\sqrt{3}}{3}$$

$$0 \leq 3x < 6\pi$$

$$\text{Add } \frac{12\pi}{6}$$

$$3x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}$$

$$x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$$