

I. Simplify each expression.

1.  $\csc \theta - \cos \theta \cot \theta$

$$\frac{1}{\sin \theta} - \cos \theta \left( \frac{\cos \theta}{\sin \theta} \right)$$

$$\frac{1}{\sin \theta} - \frac{\cos^2 \theta}{\sin \theta}$$

$$\frac{1 - \cos^2 \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta}{\sin \theta}$$

$$\boxed{\sin \theta}$$

4.  $\frac{1 + \sin^2 \theta - \cos^2 \theta}{\sec^2 \theta}$

$$\frac{\sin^2 \theta + 1 - \cos^2 \theta}{\sec^2 \theta}$$

$$\frac{\sin^2 \theta + \sin^2 \theta}{\sec^2 \theta}$$

$$\frac{2 \sin^2 \theta}{\sec^2 \theta}$$

$$\frac{2 \sin^2 \theta}{1/\cos^2 \theta}$$

$$2 \sin^2 \theta \cdot \cos^2 \theta$$

$$\boxed{2 \sin^2 \theta \cos^2 \theta}$$

2.  $\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} \cdot \frac{(1 - \sin \theta)}{(1 - \sin \theta)}$

$$\frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)} + \frac{1 - 2 \sin \theta + \sin^2 \theta}{\cos \theta (1 - \sin \theta)}$$

$$\frac{\cos^2 \theta + \sin^2 \theta + 1 - 2 \sin \theta}{\cos \theta (1 - \sin \theta)}$$

$$\frac{1 + 1 - 2 \sin \theta}{\cos \theta (1 - \sin \theta)}$$

$$\frac{2 - 2 \sin \theta}{\cos \theta (1 - \sin \theta)}$$

$$\frac{2(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)} = \frac{2}{\cos \theta} = \boxed{2 \sec \theta}$$

5.  $\frac{-\sec(x)}{\csc(-x)}$

$$\frac{-\sec x}{-\csc x}$$

$$\frac{-1/\cos x}{-1/\sin x}$$

$$\frac{-1}{\cos x} \cdot \frac{-\sin x}{1}$$

$$\frac{\sin x}{\cos x}$$

$$\boxed{\tan x}$$

3.  $\tan \alpha (\cot \alpha + \tan \alpha)$

$$\tan \alpha \cot \alpha + \tan^2 \alpha$$

$$\tan \alpha \left( \frac{1}{\tan \alpha} \right) + \tan^2 \alpha$$

$$1 + \tan^2 \alpha$$

$$\boxed{\sec^2 \alpha}$$

6.  $\csc^2 \theta - \sin^2 \theta - \cos^2 \theta - \cot^2 \theta$

$$\cot^2 + 1 - \sin^2 \theta - (1 - \sin^2 \theta) - \cot^2$$

$$\cot^2 + 1 - \sin^2 \theta - 1 + \sin^2 \theta - \cot^2$$

$$\boxed{0}$$

II. Solve each equation on the interval  $[0, 2\pi)$ .

7.  $9 \tan^2 \theta - 1 = 6 \tan^2 \theta$   
 $\frac{-6 \tan^2 \theta}{-6 \tan^2 \theta} \quad \frac{-6 \tan^2 \theta}{-6 \tan^2 \theta}$

$$3 \tan^2 \theta - 1 = 0$$

$$3 \tan^2 \theta = 1$$

$$\sqrt{\tan^2 \theta} = \sqrt{\frac{1}{3}}$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\tan \theta = \pm \frac{\sqrt{3}}{3} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

10.  $\sin \alpha \cos \alpha = \sin \alpha$

$$\sin \alpha \cos \alpha - \sin \alpha = 0$$

$$\sin \alpha (\cos \alpha - 1) = 0$$

$$\sin \alpha = 0 \quad \cos \alpha - 1 = 0$$

$$\cos \alpha = 1$$

$$\alpha = 0, \pi$$

8.  $\sin^2 \theta - 1 = 0$   
 $\sqrt{\sin^2 \theta} = \sqrt{1}$

$$\sin \theta = \pm 1$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

9.  $5 \csc x + 10 = 0$

$$5 \csc x = -10$$

$$\csc x = -2$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

11.  $4 \cos^2 x + 2 = 3$

$$4 \cos^2 x = 1$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{1}{4}}$$

$$\cos x = \pm \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

12.  $5 - 4 \cos^2 \theta = 4 \sin \theta$

$$5 - 4(1 - \sin^2 \theta) = 4 \sin \theta$$

$$5 - 4 + 4 \sin^2 \theta = 4 \sin \theta$$

$$4 \sin^2 \theta - 4 \sin \theta + 1 = 0$$

$$(4 \sin^2 \theta - 2 \sin \theta) - (2 \sin \theta - 1) = 0$$

$$2 \sin \theta (2 \sin \theta - 1) - 1(2 \sin \theta - 1) = 0$$

$$(2 \sin \theta - 1)(2 \sin \theta - 1) = 0$$

$$2 \sin \theta - 1 = 0 \quad 2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

13.  $\sec x - 2 \tan x = 0$

$$\frac{1}{\cos x} - 2 \left( \frac{\sin x}{\cos x} \right) = 0$$

$$\frac{1}{\cos x} - \frac{2 \sin x}{\cos x} = 0$$

$$\frac{1 - 2 \sin x}{\cos x} = 0 \cdot \cos x$$

$$1 - 2 \sin x = 0$$

$$-2 \sin x = -1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

14.  $\frac{1}{\sec \theta - 1} - \frac{1}{\sec \theta + 1} = 2$

$$\frac{\sec \theta + 1}{(\sec \theta + 1)(\sec \theta - 1)} - \frac{\sec \theta - 1}{(\sec \theta + 1)(\sec \theta - 1)} = 2$$

$$\frac{2}{\sec^2 \theta - 1} = 2$$

$$2 \sec^2 \theta - 2 = 2$$

$$2 \sec^2 \theta = 4$$

$$\sec^2 \theta = 2$$

$$\sqrt{\cos^2 \theta} = \sqrt{\frac{1}{2}}$$

$$\cos \theta = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

15.  $3 \tan \frac{\theta}{2} + 3 = 0$

$$3 \tan \frac{\theta}{2} = -3$$

$$\tan \frac{\theta}{2} = -1$$

Interval:  $0 \leq \frac{\theta}{2} < \pi$

$$\frac{\theta}{2} = \frac{3\pi}{4} \cdot 2$$

$$\theta = \frac{3\pi}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$