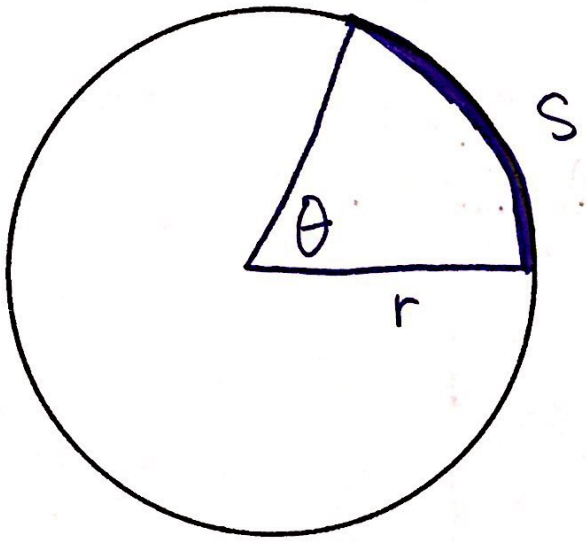
Radian Measure

One radian is the measure of a central angle Θ that intercepts an arc s equal in length to the radius r of the circle.



arc length, radius

$S = r$

$\Theta = 1$ radian

Central angle

In other words, you can think of the central angle as "how many radii would it take to make up the arc length."

A radian is defined as the angle equal to the ratio of the arc length to the radius

$$\Theta = \frac{S}{r}$$

One revolution around a circle of radius r corresponds to 2π radians because

$$\text{Circumference} = 2\pi r$$

So, for an arc length of the entire circle (the circumference),

$$\Theta = \frac{S}{r} = \frac{C}{r} = \frac{2\pi r}{r} = 2\pi$$

Thus, there are 2π radians around a circle.

Using this formula for radians allows us to solve any problem that requires us to find the central angle, radius or arc length.

$$\Theta = \frac{S}{r} \qquad S = \Theta r \qquad r = \frac{S}{\Theta}$$

We will use this tomorrow...

Converting Degrees to Radians and Radians to Degrees

Since there are 2π radians in a circle, and a circle consists of 360° we can conclude that the two angle measurements are equivalent (but different units). $2\pi = 360^\circ$

Thus, in order to convert from Degree measure to Radian measure (and vice versa), you simply need to remember one simple rule:

360 degrees is the same as 2π radians, and hence, 180 degrees is the same as π radians.

$180^\circ = \pi$ radians, so, $\frac{180^\circ}{\pi} = 1$ OR $\frac{\pi}{180^\circ} = 1$

Convert from degrees to radians:

Multiply by $\frac{\pi}{180^\circ}$

Convert from radians to degrees:

Multiply by $\frac{180^\circ}{\pi}$

Ex: Convert from degrees to radians, or radians to degrees.

1. $240^\circ \cdot \frac{\pi}{180} = \frac{240\pi}{180} = \boxed{\frac{4\pi}{3}}$

2. $\frac{3\pi}{4}$ radians $\cdot$

$\frac{3\pi}{4} \cdot \frac{180}{\pi} = \frac{540}{4} = \boxed{135^\circ}$

Sketch each angle in standard position.

1. $\frac{\pi}{4}$
2. $\frac{5\pi}{6}$
3. $-\frac{3\pi}{4}$
4. $\frac{5\pi \cdot 2}{3 \cdot 2}$ $\frac{10\pi}{6}$
5. $\frac{11\pi}{6}$
6. 4

$\pi < 4 < \frac{3\pi}{2}$

$3.14 < 4 < 4.71$