Pre-Calculus
Name: $\qquad$
Spring Final Exam Review

## Solve

1. $5+2 \sin x-7=0$
2. $4 \cos ^{2} x-3=0$
3. $\sin x \cos x-5 \cos x=0$
4. $3 \cos x=\cos x-1$
5. $\sin ^{2} x=1-\cos x$
6. $\tan ^{3} x=-\tan ^{2} x$

## Simplify

7. $\sec ^{2} x \tan ^{2} x+\sec ^{2} x$
8. $(\csc x+1)(\csc x-1)$
9. $\csc x \tan ^{2} x-\sec ^{2} x \csc x$
10. $\frac{\tan ^{2} x+1}{1-\sin ^{2} x}$
11. $\frac{1+\tan ^{2} x}{\csc x \sec x}$
12. $\frac{1-\sin ^{4} x}{1+\sin ^{2} x}$
13. $\frac{\sec x}{\tan x}$
14. $\frac{1-\cot ^{4} x}{1-\cot ^{2} x}$
15. $\cos \left(\frac{\pi}{2}-x\right) \sec x$
16. $\cot \left(\frac{\pi}{2}-x\right) \cos x$

## Prove the following are identities:

17. $(\sin x+\cos x)^{2}+(\sin x-\cos x)^{2}=2$
18. $\csc ^{4} x-2 \csc ^{2} x+1=\cot ^{4} x$
19. $\tan x(\csc x-\sin x)=\cos x$
20. $\frac{\cos ^{4} x-\sin ^{4} x}{\cos 2 x}=1$
21. $\csc 2 x=\frac{1}{2} \csc x \sec x$
22. $\frac{\sec ^{2} x}{\tan x}=\sec x \csc x$
23. $\sec x-\sin x \tan x=\cos x$
24. $\sec ^{2}\left(\frac{\pi}{2}-x\right)-1=\cot ^{2} x$
25. $\frac{\csc ^{2} x}{\csc ^{2} x-1}=\sec ^{2} x$
26. $\frac{1+\tan ^{2} x}{\cos ^{2} x}=\sec ^{4} x$

## Solve the triangle

27. $B=38^{\circ} ; C=21^{\circ} ; b=24$
28. $C=61^{\circ} ; c=30 ; b=25$
29. $a=20 ; b=10 ; A=31^{\circ}$
30. $a=12 ; b=14 ; c=20$
31. $\mathrm{B}=72^{\circ} ; \mathrm{a}=78 ; \mathrm{c}=16$
32. $A=70^{\circ} ; B=12^{\circ} ; b=21$

Find the area of the triangle -
33. $B=42^{\circ} ; a=7 ; c=3$
34. $C=75^{\circ} ; a=1.5 ; b=2$
35. $A=113^{\circ} ; b=18 ; c=24$

## Find the missing information

36. Given $a=4, b=5$, and $c=2.5$, find angle $B$
37. Triangle has sides $a=15, b=20$, and $A=29^{\circ}$, find the rest!
38. Triangle has sides $a=15, b=28$, and $A=44^{\circ}$, find the rest!
39. Triangle $A B C$ where points $A$ and $B$ are opposite points of a crater. Point $C$ is 50 m from point $A$. If $A$ is $112^{\circ}$ and $C$ is $42^{\circ}$. What is the length between $A$ and $B$ (the width of the crater)?
40. A sail on a boat is in the shape of an isosceles triangle and the top vertex angle is $38^{\circ}$. The two sides come down at 20 ft . each. What are the base angle measurements and what is the length of the base itself?
41. From the top of a 200 -ft lighthouse, the angle of depression to a ship on the ocean is $23^{\circ}$. How far is the ship from the base of the lighthouse?
42. A 96-ft tree casts a shadow that is 120 ft long. What is the angle of elevation?
43. A man is lying on the beach, flying a kite. He holds the end of the kite string at ground level and estimates the angle of elevation to the kite to be $50^{\circ}$. If the string is 450 ft long, how high is the kite above the ground?
44. A builder wishes to construct a ramp 24 ft long that rises to a height of 5 feet above the ground. Find the angle of elevation of the ramp.

## Given the right triangle $A B C$ ( $C$ is the right angle), find all remaining sides/angles-

45. $\mathrm{A}=29^{\circ}, \mathrm{C}=11$
46. $b=3, c=7$
47. $\mathrm{B}=13^{\circ}, \mathrm{c}=6$

## Given these vectors, graph the following resultants



## Graph and label the following 3-d points (you only need one grid...)

51. (-2, 3, -6)
52. ( $3,-4,5)$
53. ( $4,-5,-3)$

Find the magnitude for the vectors -
54. $\langle 4,5\rangle$
55. $\langle-2,5,-4\rangle$
56. $\langle-10,4,7\rangle$

Given the points (initial followed by terminal), write the component form of the vector
57. $(2,-5)(9,-11)$
59. $(-3,-5,-8)(11,-12,-14)$
58. $(4,3,5) \quad(7,-1,8)$

Determine if the following vectors are orthogonal.
60. $\langle 4,6\rangle$ and $\langle 7,9\rangle$
61. $\langle 9,-9,6\rangle$ and $\langle 6,4,-3\rangle$

Find the direction angle of each vector
62. $\langle 4,7\rangle$
63. $\langle-6,-2\rangle$
64. $\langle-8,0\rangle$
65. $\langle 5,-1\rangle$

Find the angle between these vectors
66. $\langle 2,-3\rangle$ and $\langle-4,3\rangle$.
67. $\langle-4,-1,-3\rangle$ and $\langle 7,3,4\rangle$

## Solve

68. A ship is moving due west at 12 mph . The water is moving at 3 mph due north. What is the resultant of the two and what direction is the ship actually going?

Find all solutions over the interval of $[0,2 \pi)$
69. $\sec x=1+\tan x$
70. $\cos 2 x+\sin x=1$
71. $\sin \frac{\theta}{2}+\cos \theta=1$
72. $3 \tan ^{2} \theta-\sqrt{3} \tan \theta=0$

Write the first 5 terms of the defined sequence
73. $a_{n}=\frac{6 n+5}{n}$
75. $a_{n}=2048\left(\frac{1}{2}\right)^{n-1}$
74. $a_{n}=3(2)^{n-1}$
76. $a_{n}=\frac{a_{n-1}-3}{4} ; a_{1}=144$

Write an explicit rule for each sequence
77. $-13,-7,-1,5,11,17, \ldots$
78. $0.75,3,12,48,192, \ldots$
79. $-\frac{1}{64}, \frac{1}{8},-1,8,-64, \ldots$
80. An arithmetic series with $1^{\text {st }}$ term 199 and $58^{\text {th }}$ term 940

## Write a recursive rule for each

84. $3,10,17,24,31,38, \ldots$
85. $3,6,12,24,48, \ldots$

## Evaluate each series for the number of given terms

86. $2+6+18+54+\ldots ; n=6$
87. $-1+(-2)+(-4)+(-8)+\ldots ; n=7$
88. $35+29+23+17+\ldots ; n=15$
89. $27+37+47+57+\ldots ; n=16$

## Evaluate each series

90. $\sum_{n=1}^{9} 3(4)^{n-1}$
91. $\sum_{m=13}^{52}(2.3 m+4.5)$
92. $\sum_{k=1}^{15}(6-7 k)$
93. $\sum_{i=1}^{\infty} 8(-.5)^{i-1}$
94. $\sum_{x=1}^{40}(15-9 x)$
95. $\sum_{d=1}^{9}(-5)^{d-1}$
96. $\sum_{n=4}^{13}\left(-\frac{31}{15}+\frac{2}{3} n\right)$
97. $\sum_{j=1}^{\infty} 8(-2)^{j-1}$
98. A geometric series with $1^{\text {st }}$ term 14 and $5^{\text {th }}$ term 70.875
99. An arithmetic series with $23^{\text {rd }}$ term 125 and $111^{\text {th }}$ term -403
100. A geometric series with $11^{\text {th }}$ term 4 and $22^{\text {nd }}$ term 8192
