Pre-Calculus

Spring Final Exam Review

Solve

- 1. $5 + 2 \sin x 7 = 0$
- 2. $4\cos^2 x 3 = 0$
- 3. $\sin x \cos x 5 \cos x = 0$

Simplify

7. $\sec^2 x \tan^2 x + \sec^2 x$ 8. $(\csc x + 1) (\csc x - 1)$ 9. $\csc x \tan^2 x - \sec^2 x \csc x$ 10. $\frac{\tan^2 x + 1}{1 - \sin^2 x}$ 11. $\frac{1 + \tan^2 x}{\csc x \sec x}$ 12. $\frac{1 - \sin^4 x}{1 + \sin^2 x}$ 4. $3\cos x = \cos x - 1$

Name: _____

- $5. \quad \sin^2 x = 1 \cos x$
- $6. \quad tan^3x = -tan^2x$

13.
$$\frac{\sec x}{\tan x}$$

14.
$$\frac{1 - \cot^4 x}{1 - \cot^2 x}$$

15.
$$\cos\left(\frac{\pi}{2} - x\right) \sec x$$

16.
$$\cot\left(\frac{\pi}{2} - x\right) \cos x$$

23. $\frac{\sec^2 x}{\tan x} = \sec x \csc x$

 $25.\frac{csc^2x}{csc^2x-1} = sec^2x$

 $26. \frac{1+tan^2x}{cos^2x} = sec^4x$

22. $csc^4x - 2csc^2x + 1 = cot^4x$

24. $\sec x - \sin x \tan x = \cos x$

Prove the following are identities:

17.
$$(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$$

18. $\tan x (\csc x - \sin x) = \cos x$
19. $\frac{\cos^4 x - \sin^4 x}{\cos 2x} = 1$
20. $\csc 2x = \frac{1}{2}\csc x \sec x$
21. $\sec^2(\frac{\pi}{2} - x) - 1 = \cot^2 x$

Solve the triangle

27.
$$B = 38^{\circ}$$
; $C = 21^{\circ}$; $b = 24$ 30. $a = 12$; $b = 14$; $c = 20$ 28. $C = 61^{\circ}$; $c = 30$; $b = 25$ 31. $B = 72^{\circ}$; $a = 78$; $c = 16$ 29. $a = 20$; $b = 10$; $A = 31^{\circ}$ 32. $A = 70^{\circ}$; $B = 12^{\circ}$; $b = 21$

Find the area of the triangle -

Find the missing information

- 36. Given a = 4, b = 5, and c = 2.5, find angle B
- 37. Triangle has sides a = 15, b = 20, and A = 29°, find the rest!
- 38. Triangle has sides a = 15, b = 28, and A = 44°, find the rest!
- 39. Triangle ABC where points A and B are opposite points of a crater. Point C is 50 m from point A. If A is 112° and C is 42°. What is the length between A and B (the width of the crater)?
- 40. A sail on a boat is in the shape of an isosceles triangle and the top vertex angle is 38°. The two sides come down at 20 ft. each. What are the base angle measurements and what is the length of the base itself?
- 41. From the top of a 200-ft lighthouse, the angle of depression to a ship on the ocean is 23°. How far is the ship from the base of the lighthouse?
- 42. A 96-ft tree casts a shadow that is 120 ft long. What is the angle of elevation?
- 43. A man is lying on the beach, flying a kite. He holds the end of the kite string at ground level and estimates the angle of elevation to the kite to be 50°. If the string is 450 ft long, how high is the kite above the ground?
- 44. A builder wishes to construct a ramp 24 ft long that rises to a height of 5 feet above the ground. Find the angle of elevation of the ramp.

Given the right triangle ABC (C is the right angle), find all remaining sides/angles-

45. A = 29°, c = 11
46. b = 3, c = 7
47. B = 13°, c = 6

Given these vectors, graph the following resultants



Graph and label the following 3-d points (you only need one grid...)

51. (-2, 3, -6) 52. (3, -4, 5) 53. (4, -5, -3)

Find the magnitude for the vectors -

54. (4,5) 55. (-2,5,-4) 56. (-10,4,7)

Given the points (initial followed by terminal), write the component form of the vector

57. (2, -5) (9, -11)59. (-3, -5, -8) (11, -12, -14)58. (4, 3, 5) (7, -1, 8)

Determine if the following vectors are orthogonal.

60. (4, 6) and (7, 9) 61. (9, -9, 6) and (6, 4, -3)

Find the direction angle of each vector

$$62. \langle 4, 7 \rangle$$
 $64. \langle -8, 0 \rangle$
 $63. \langle -6, -2 \rangle$
 $65. \langle 5, -1 \rangle$

Find the angle between these vectors

66. (2, -3) and (-4, 3). 67. (-4, -1, -3) and (7, 3, 4)

Solve

68. A ship is moving due west at 12 mph. The water is moving at 3 mph due north. What is the resultant of the two and what direction is the ship actually going?

Find all solutions over the interval of $\left[0,2\pi ight)$

69. sec x = 1 + tan x 70. cos 2x + sin x = 1 71. sin $\frac{\theta}{2}$ + cos θ = 1 72. $3 \tan^2 \theta - \sqrt{3} \tan \theta = 0$

Write the first 5 terms of the defined sequence

73.
$$a_n = \frac{6n+5}{n}$$

74. $a_n = 3(2)^{n-1}$
75. $a_n = 2048 \left(\frac{1}{2}\right)^{n-1}$
76. $a_n = \frac{a_{n-1}-3}{4}; a_1 = 144$

Write an explicit rule for each sequence

77. -13, -7, -1, 5, 11, 17, . . . 78. 0.75, 3, 12, 48, 192, . . .

79.
$$-\frac{1}{64}, \frac{1}{8}, -1, 8, -64, \dots$$

 An arithmetic series with 1st term 199 and 58th term 940

Write a recursive rule for each

- 84. 3, 10, 17, 24, 31, 38, . . .
- 85. 3, 6, 12, 24, 48, . . .

Evaluate each series for the number of given terms

86. 2 + 6 + 18 + 54 + ...; n = 6
87. -1 + (-2) + (-4) + (-8) + ...; n = 7
88. 35 + 29 + 23 + 17 + ...; n = 15
89. 27 + 37 + 47 + 57 + ...; n = 16

Evaluate each series

90.
$$\sum_{n=1}^{9} 3(4)^{n-1}$$

91.
$$\sum_{k=1}^{15} (6-7k)$$

92.
$$\sum_{i=1}^{\infty} 8(-.5)^{i-1}$$

93.
$$\sum_{x=1}^{40} (15-9x)$$

94.
$$\sum_{d=1}^{9} (-5)^{d-1}$$

95.
$$\sum_{n=4}^{13} \left(-\frac{31}{15} + \frac{2}{3}n\right)$$

96.
$$\sum_{j=1}^{\infty} 8(-2)^{j-1}$$

- A geometric series with 1st term 14 and 5th term 70.875
- 82. An arithmetic series with 23rd term 125 and 111th term -403
- A geometric series with 11th term 4 and 22nd term 8192

97.
$$\sum_{m=13}^{52} (2.3m + 4.5)$$