Simplifying Expressions

RECIPROCAL IDENTITIES

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

QUOTIENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

$\sin^2\theta = 1 - \cos^2\theta$

Simplify.

1. $\cot\theta\sec\theta\sin\theta$

$\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{1}\right)$

1

2. $(1-\sin x)(1+\sin x)$ FOIL

$1 + \sin x - \sin x - \sin^2 x$

$1 - \sin^2 x$

$\cos^2 x$

$\sin^2 x + \cos^2 x = 1$

$-\sin^2 x \qquad -\sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

3. $\cos x(\sec x - \cos x)$

$\cos x\left(\frac{1}{\cos x} - \cos x\right)$

$1 - \cos^2 x$

$\sin^2 x$

4. $\sin^2\theta + \cos^2\theta + \tan^2\theta$

$1 + \tan^2\theta$

$\sec^2\theta$

5. $\dfrac{\sin x \cos x}{1-\cos^2 x}$

$\dfrac{\sin x \cos x}{\sin^2 x}$

$\dfrac{\cos x}{\sin x}$

$\boxed{\cot x}$

6. $\csc\theta \tan\theta$

$\left(\dfrac{1}{\sin\theta}\right)\left(\dfrac{\sin\theta}{\cos\theta}\right)$

$\dfrac{1}{\cos\theta}$

$\boxed{\sec\theta}$

7. $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x}$

$\cos x \cdot \dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} \cdot \dfrac{1+\sin x}{1+\sin x}$

$\dfrac{\cos^2 x}{\cos x(1+\sin x)} + \dfrac{1+\sin x+\sin x+\sin^2 x}{\cos x(1+\sin x)}$

$\dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1+\sin x)}$

$1 = \dfrac{\cos^2 x + \sin^2 x + 1 + 2\sin x}{\cos x(1+\sin x)}$

$\dfrac{1+1+2\sin x}{\cos x(1+\sin x)}$

$\dfrac{2+2\sin x}{\cos x(1+\sin x)} = \dfrac{2(1+\sin x)}{\cos x(1+\sin x)} = \dfrac{2}{\cos x} = \boxed{2\sec x}$

HW: p. 317: 22-34, 36