

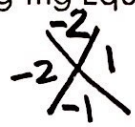
$$\cos^2 x = 1 - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

Solving Trig Equations with Substitution or Rewriting

Ex: Solve for $0 \leq \theta \leq 2\pi$

1. $2\cos^2 x - \sin x - 1 = 0$



$$2(1 - \sin^2 x) - \sin x - 1 = 0$$

$$2 - 2\sin^2 x - \sin x - 1 = 0$$

$$-2\sin^2 x - \sin x + 1 = 0$$

$$(-2\sin^2 x - 2\sin x) + (\sin x + 1) = 0$$

$$-2\sin x(\sin x + 1) + 1(\sin x + 1) = 0$$

$$(\sin x + 1)(-2\sin x + 1) = 0$$

$$\sin x + 1 = 0 \quad -2\sin x + 1 = 0$$

$$\sin x = -1$$

$$-2\sin x = -1$$

$$\sin x = \frac{1}{2}$$

$$\boxed{\frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$$

3. $(\sec x + 1)^2 = (\tan x)^2$

$$(\sec x + 1)(\sec x + 1) = \tan^2 x$$

$$\sec^2 x + \sec x + \sec x + 1 = \tan^2 x$$

$$\sec^2 x + 2\sec x + 1 = \tan^2 x$$

$$\cancel{\sec^2 x} + 2\sec x + 1 = \cancel{\sec^2 x} - 1 + 1$$

$$2\sec x + 2 = 0$$

$$2\sec x = -2$$

$$\sec x = -1$$

$$\cos x = -1$$

$$\boxed{\pi}$$

2. $1 - \cos x = 2\sin^2 x$

$$1 - \cos x = 2(1 - \cos^2 x)$$

$$1 - \cos x = 2 - 2\cos^2 x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos^2 x - 2\cos x) + (\cos x - 1) = 0$$

$$2\cos x(\cos x - 1) + 1(\cos x - 1) = 0$$

$$(\cos x - 1)(2\cos x + 1) = 0$$

$$\cos x - 1 = 0 \quad 2\cos x + 1 = 0$$

$$\cos x = 1$$

$$2\cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$$\boxed{0\pi, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}}$$

4. $\cot^2 x \csc^2 x + 2\csc^2 x - \cot^2 x = 2$

$$(\cot^2 x \csc^2 x + 2\csc^2 x) - (\cot^2 x - 2) = 0$$

$$\csc^2 x(\cot^2 x + 2) - 1(\cot^2 x + 2) = 0$$

$$(\cot^2 x + 2)(\csc^2 x - 1) = 0$$

$$\cot^2 x + 2 = 0$$

$$\csc^2 x - 1 = 0$$

$$\cot^2 x = -2$$

$$\csc^2 x = 1$$

$$\sqrt{\tan^2 x} = \sqrt{-\frac{1}{2}}$$

$$\sqrt{\sin^2 x} = \sqrt{1}$$

$$\sin x = \pm 1$$

$$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$$