Sum/Difference Formulas

Sum:

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ Susan Can't Come Saturday

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ Carol Can't Stand Susan

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

Difference:

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

**Angles not on the unit circle.

Ex: Find the exact value.

1. $\cos 165°$

$\cos(135+30)$

$= \cos 135\cos 30 - \sin 135\sin 30$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

2. $\sin\left(\frac{21\pi}{12}\right)$

$\sin\left(\frac{7\pi}{4}\right)$

$\boxed{-\frac{\sqrt{2}}{2}}$

3. $\tan\left(\frac{25\pi}{12}\right)$

$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}$

$\frac{2\pi}{12}, \frac{3\pi}{12}, \frac{4\pi}{12}, \frac{6\pi}{12}, \frac{8\pi}{12}, \frac{9\pi}{12}, \frac{10\pi}{12}, \frac{12\pi}{12}, \frac{14\pi}{12}, \frac{15\pi}{12}$

$\tan\left(\frac{10\pi}{12} + \frac{15\pi}{12}\right) = \tan\left(\frac{5\pi}{6} + \frac{5\pi}{4}\right)$

$= \dfrac{\tan\frac{5\pi}{6} + \tan\frac{5\pi}{4}}{1 - \tan\frac{5\pi}{6}\tan\frac{5\pi}{4}} = \dfrac{-\frac{\sqrt{3}}{3} + 1}{1 - \left(-\frac{\sqrt{3}}{3}\right)(1)} = \dfrac{3\left(-\frac{\sqrt{3}}{3}\right) + 1(3)}{3(1) + \left(\frac{\sqrt{3}}{3}\right)3} = \dfrac{-\sqrt{3}+3}{3+\sqrt{3}}$

$= \dfrac{-\sqrt{3}+3}{3+\sqrt{3}} \cdot \dfrac{(3-\sqrt{3})}{(3-\sqrt{3})} = \dfrac{-3\sqrt{3}+3+9-3\sqrt{3}}{9-3} = \dfrac{12-6\sqrt{3}}{6} = \boxed{2-\sqrt{3}}$

Find the exact value of the expression.

4. $\dfrac{\tan 78° - \tan 18°}{1 + \tan 78° \tan 18°}$

$\tan(78-18)$

$\tan(60)$

$\boxed{\sqrt{3}}$

5. $\cos\frac{7\pi}{8}\cos\frac{5\pi}{24} + \sin\frac{7\pi}{8}\sin\frac{5\pi}{24}$

$\cos\left(\frac{7\pi}{8}\cdot\frac{3}{3} - \frac{5\pi}{24}\right)$

$\cos\left(\frac{21\pi}{24} - \frac{5\pi}{24}\right)$

$\cos\left(\frac{16\pi}{24}\right)$

$\cos\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2}}$

Find the solution to each expression on the interval [0, 2π].

7. $\cos(x+\pi) - \sin(x-\pi) = 0$

$\cos x\cos\pi - \sin x\sin\pi - (\sin x\cos\pi - \cos x\sin\pi) = 0$

$\cos x(-1) - \sin x(0) - (\sin x(-1) - \cos x(0)) = 0$

$-\cos x - (-\sin x) = 0$

$-\cos x + \sin x = 0$

$\boxed{\frac{\pi}{4} \text{ \& } \frac{5\pi}{4}}$

$\frac{\pi}{4}: -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0$ ✓

$\frac{3\pi}{4}: +\left(+\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}$

$\frac{5\pi}{4}: +\left(+\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right) = 0$ ✓

$\frac{7\pi}{4}: -\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}$

8. $\cos\left(\frac{\pi}{3}+x\right) + \cos\left(\frac{\pi}{3}-x\right) = \frac{1}{2}$

$\cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x + \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x = \frac{1}{2}$

$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$\boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$

p. 341: 1-6, 11-16, 40-45