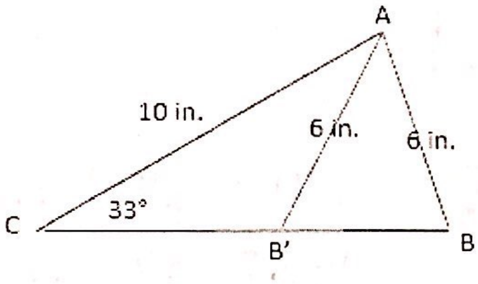
SSA--Side Side Angle

Given two sides and a non-included angle of a triangle, you might not be able to determine what type of triangle it is, or even if those pieces form a triangle at all.

Furthermore, as a result, you might have to analyze the situation before applying the law of sines and solving for the missing pieces.

For example, take a look at this picture:



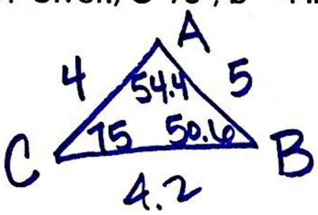
If you are told that $m\angle C = 33°$, b = 10 in. and c= 6 in, there are two different triangles that match this criteria. As you can see in the picture, either an acute triangle or an obtuse triangle could be created because side c could swing either in or out along the unknown side a.

When you are given two sides and an angle not in between those sides, you need to be on the lookout for the ambiguous case.

To determine if there is a 2nd valid angle:

1. See if you are given two sides and the angle not in between (SSA). This is the situation that may have 2 possible answers.
2. Find the value of the unknown angle using the law of sines.
3. Once you find the value of your angle, subtract it from 180° to find the possible second angle.
4. Add the new (second) angle to the original angle. If their sum is less than 180°, you have two valid answers. If the sum is over 180°, then the second angle is not valid.

1. **Given, C=75°, b = 4 in, c = 5 in**



$$\frac{\sin B}{4} = \frac{\sin 75}{5}$$

$$\frac{5\sin B}{5} = \frac{4\sin 75}{5}$$

$$\sin B = 0.7727$$

$$\boxed{B = 50.6°}$$

$$180 - 50.6 = 129.4°$$

$$129.4 \cancel{+75} = 204.4$$

1 triangle

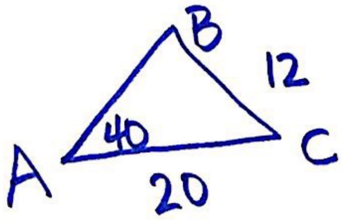
$$\angle A = 180 - 75 - 50.6 = 54.4°$$

$$\frac{\sin 54.4}{a} = \frac{\sin 75}{5}$$

$$\frac{5\sin 54.4}{\sin 75} = \frac{a\sin 75}{\sin 75}$$

$$\boxed{a = 4.2}$$

2. Given, A=40°, b = 20 ft, a = 12 ft



$$\frac{\sin B}{20} \times \frac{\sin 40}{12}$$

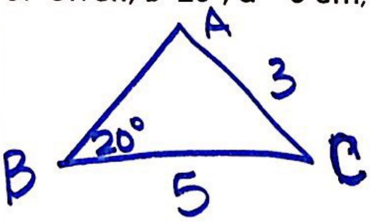
$$\frac{12 \sin B}{12} = \frac{20 \sin 40}{12}$$

$$\sin B = 1.0713$$

Error

$\boxed{\text{Not a triangle}}$

3. Given, B=20°, a = 5 cm, b = 3 cm



$$\frac{\sin A}{5} \times \frac{\sin 20}{3}$$

$$\frac{3 \sin A}{3} = \frac{5 \sin 20}{3}$$

$$\sin A = 0.5700$$

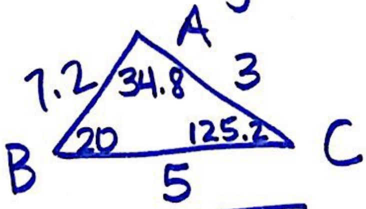
$$A = 34.8°$$

$180 - 34.8 = 145.2°$

$145.20 + 20 = 165.2°$

2 Triangles

Triangle 1



$\boxed{\angle A = 34.8°}$

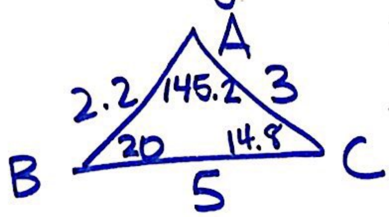
$\angle C = 180 - 34.8 - 20 = \boxed{125.2°}$

$$\frac{\sin 125.2}{c} \times \frac{\sin 20}{3}$$

$$\frac{3 \sin 125.2}{\sin 20} = \frac{c \sin 20}{\sin 20}$$

$\boxed{7.2 = c}$

Triangle 2



$\boxed{\angle A = 145.2°}$

$\angle C = 180 - 145.2 - 20 = \boxed{14.8°}$

$$\frac{\sin 14.8}{c} \times \frac{\sin 20}{3}$$

$$\frac{3 \sin 14.8}{\sin 20} = \frac{c \sin 20}{\sin 20}$$

$\boxed{2.2 = c}$