Finding sin, cos, tan of anything

What is our ultimate goal?

We want to be able to find the exact value of the sine, cosine, and tangent of certain angles.

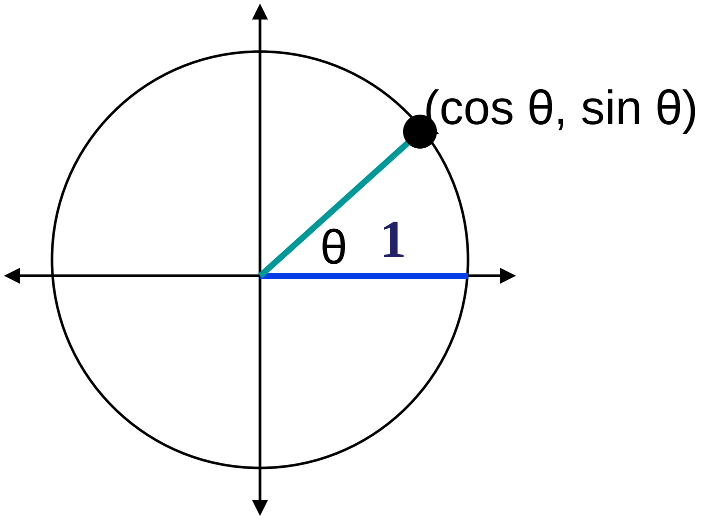
How do we do this?

1) Locate where this angle is on the unit circle.
2) Find the reference angle
3) Find the sin, cos, or tan for the reference angle.
4) Determine it's sign by its location.

That’s great if the angle we’re given is less than 90°. But what if it’s greater than 90? Or negative? Behold the unit circle!

It is a circle, with radius 1 unit, that is on the x-y coordinate plane.

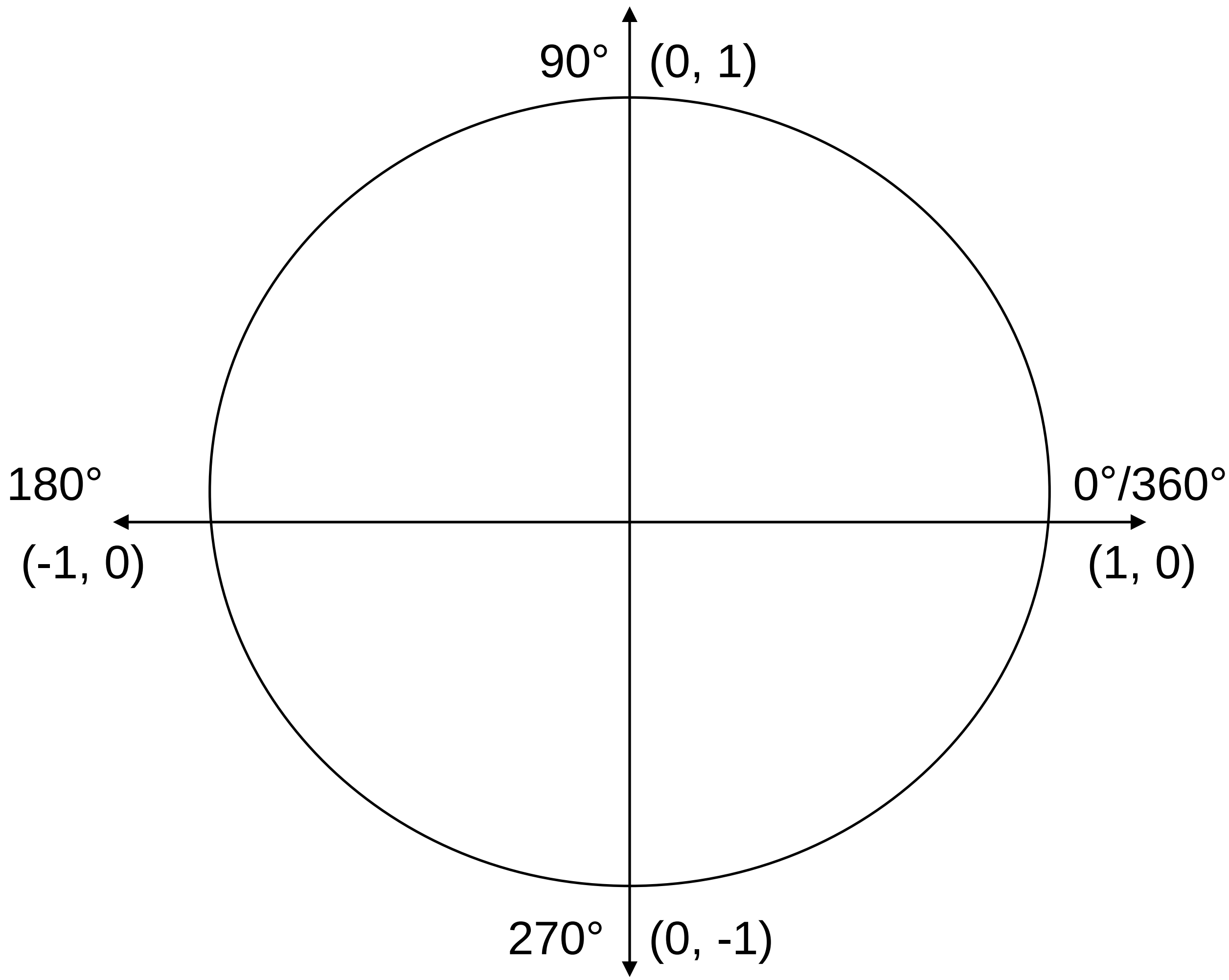
Cosine of θ: x-coordinate



Sine of θ: y-coordinate

Tangent of θ: $\frac{y}{x}$

Let's get our bearings straight on the unit circle:



(cos θ, sin θ)

Sin 90° = 1

Cos 270° = 0

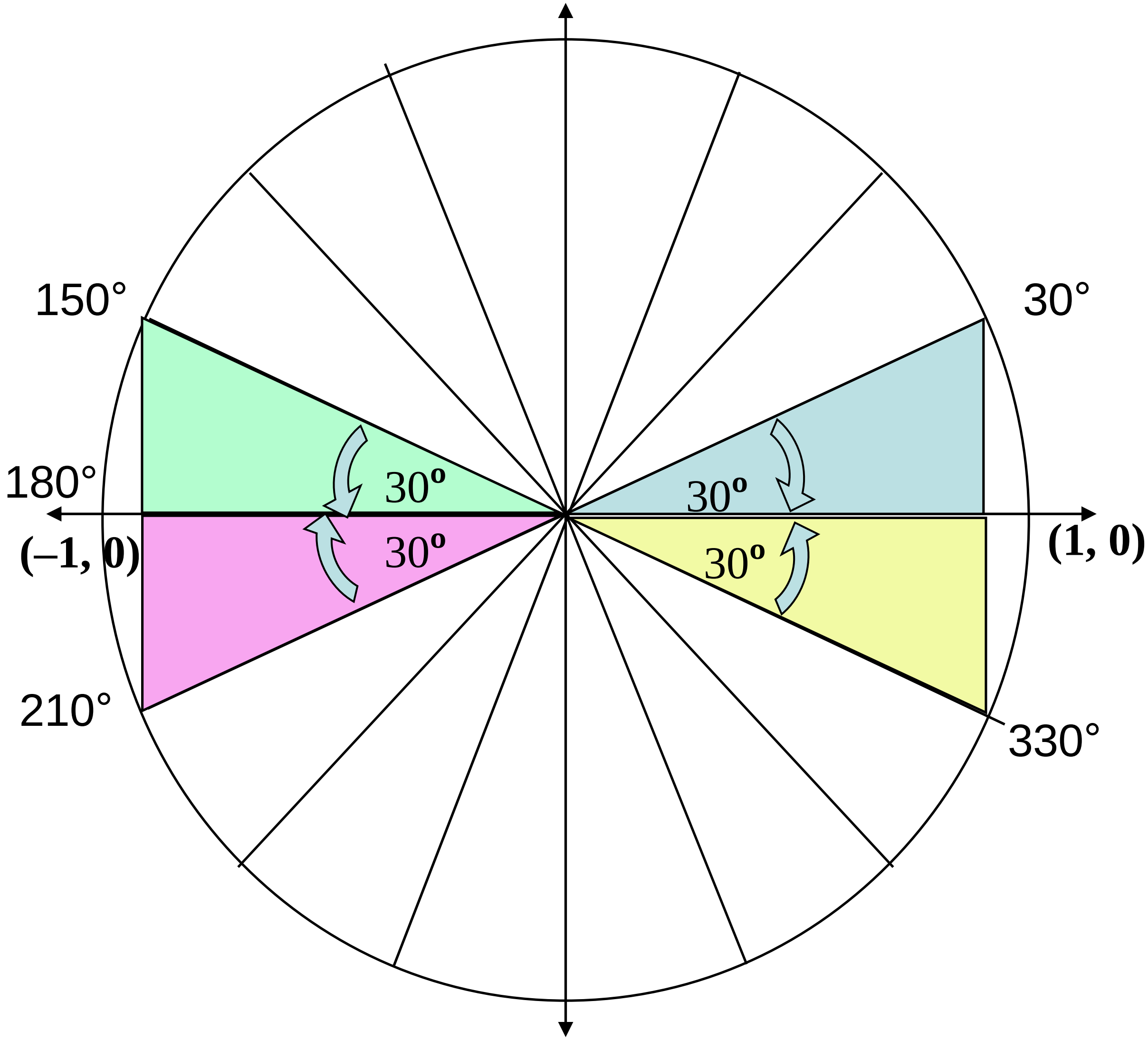
Sin 540° = Sin 180 = 0

(540-360 = 180°)

Tan 0° = $\frac{y}{x} = \frac{0}{1}$ =0

Tan 90°=$\frac{y}{x} = \frac{1}{0}$ *undefined*

Now let's look at angle measures 30, 150, 210, and 330.

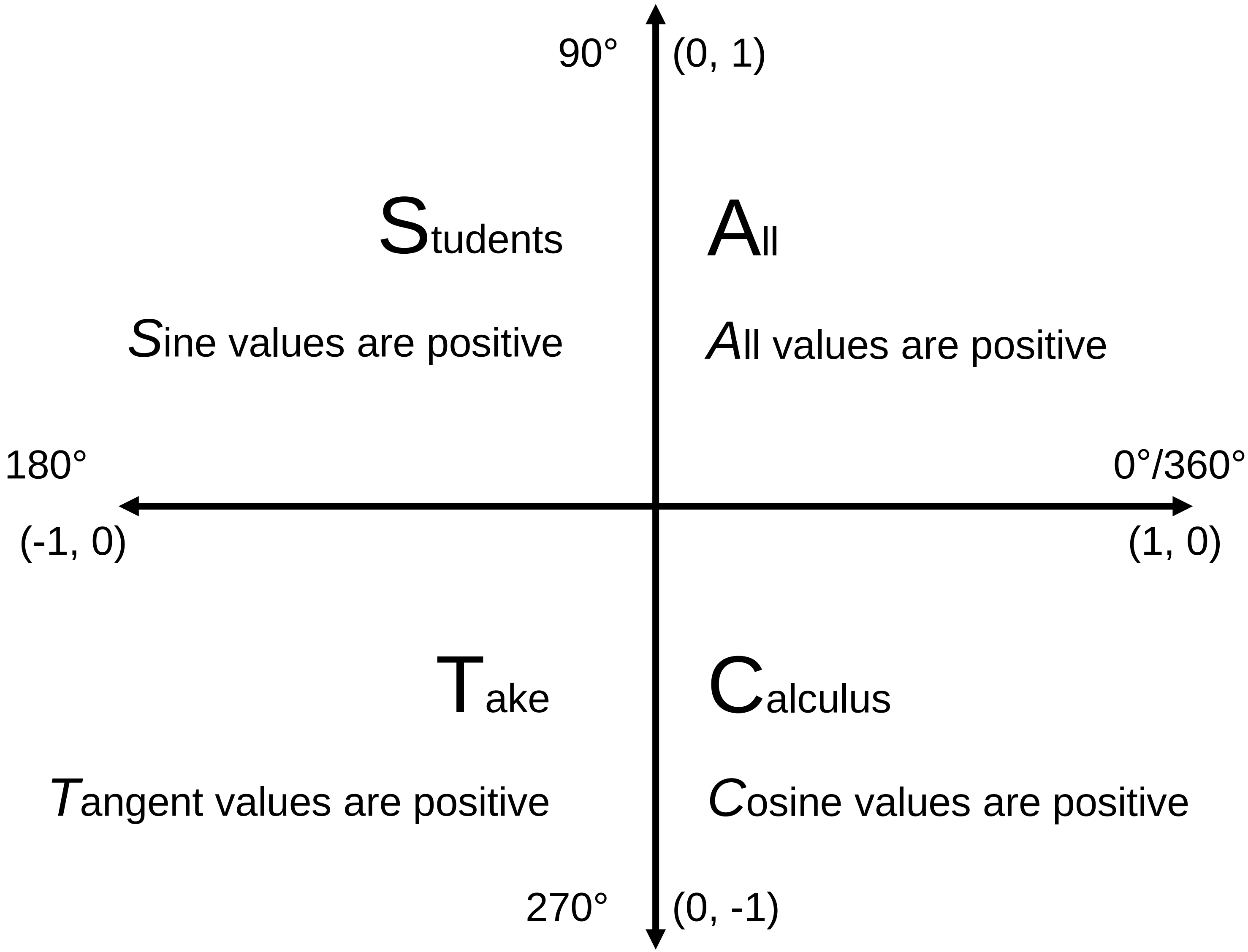


They all form a 30° angle with the x-axis, so they should all have the same sine, cosine, and tangent values…only the signs will change!

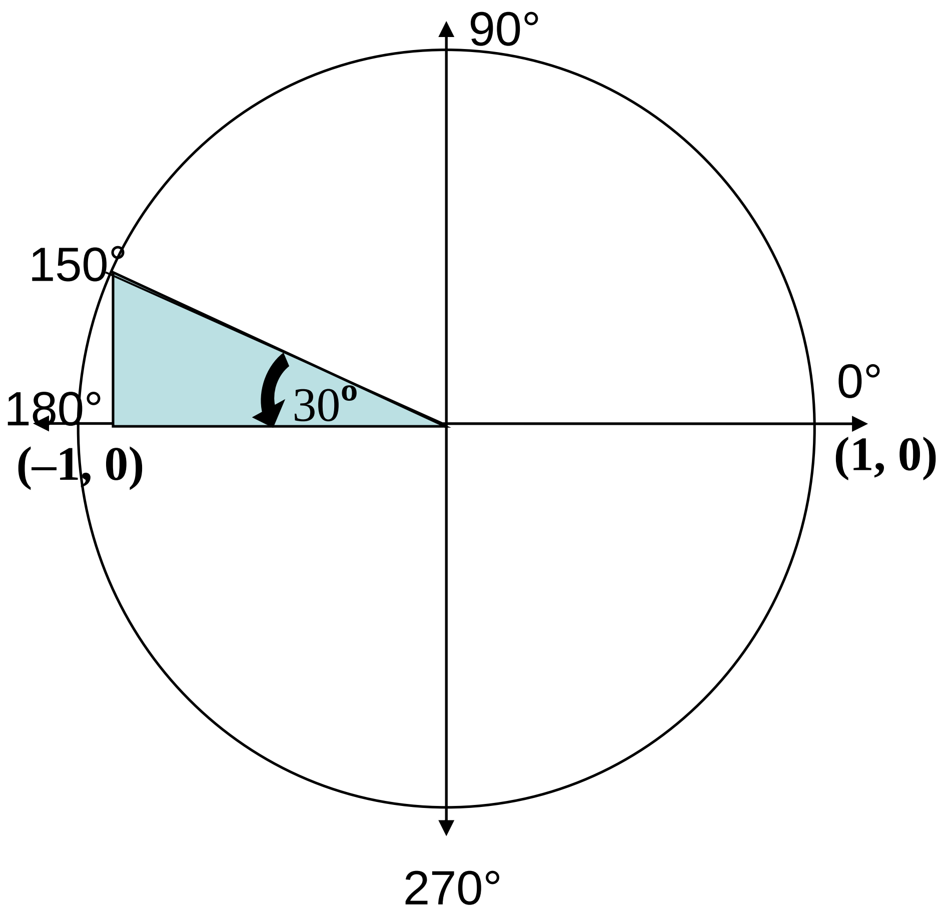
The angle to the nearest x-axis is called the *reference angle.*

All angles with the same reference angle with have the same trig values except for sign changes.

Remember….Determining sign



Example: Find the sine, cosine, and tangent values of 150°.



Where is 150°?

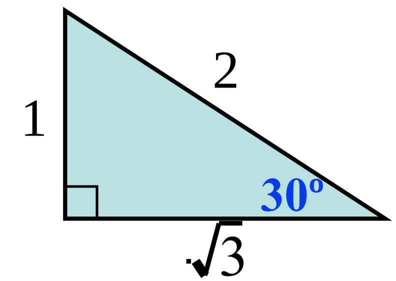
It is greater than 90, less than 180, so it is in the 2nd quadrant.

What is the reference angle?

180 – 150 = 30°

Use our 30-60-90 triangle to find values:

Sin 30° = $\frac{1}{2}$



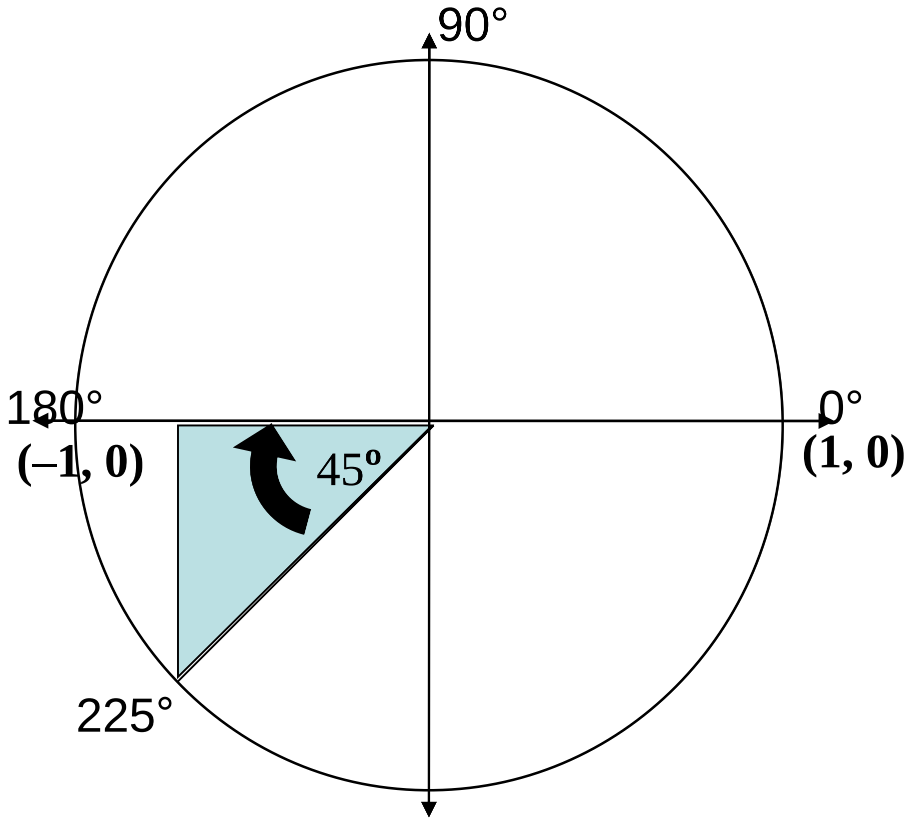
Cos 30° = $\frac{\sqrt{3}}{2}$

Tan 30° = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

It is in the second quadrant, (“students”) so only Sine is positive. So my final answers are

Sin 150° = $\frac{1}{2}$ Cos 150° = $\frac{-\sqrt{3}}{2}$ Tan 150° = $\frac{-\sqrt{3}}{3}$

Example: Find the sine, cosine, and tangent values of 225°.



Where is 225°?

It is greater than 180, less than 270, so it is in the 3rd quadrant.

What is the reference angle?

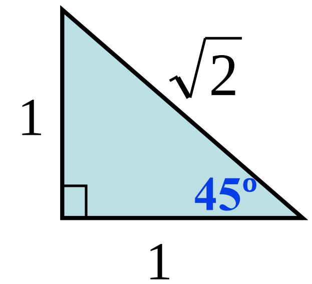
225 - 180 = 45°

Use our 45-45-90 triangle to find values:

Sin 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Cos 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

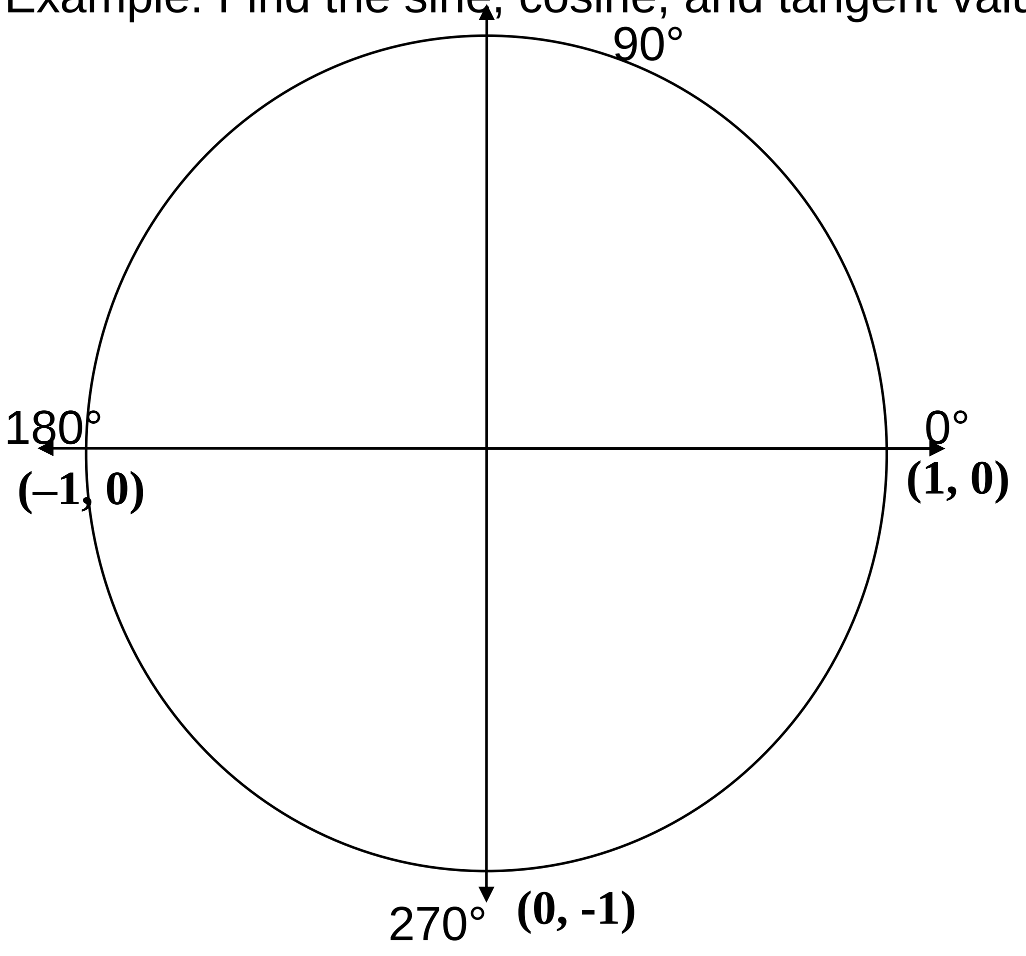
Tan 45° = $\frac{1}{1} = 1$



It is in the third quadrant, ("take") so only tangent is positive. So my final answers are

Sin 225° = $\frac{-\sqrt{2}}{2}$ Cos 225° = $\frac{-\sqrt{2}}{2}$ Tan 225° = 1

Example: Find the sine, cosine, and tangent values of 270°.



Where is 270°?

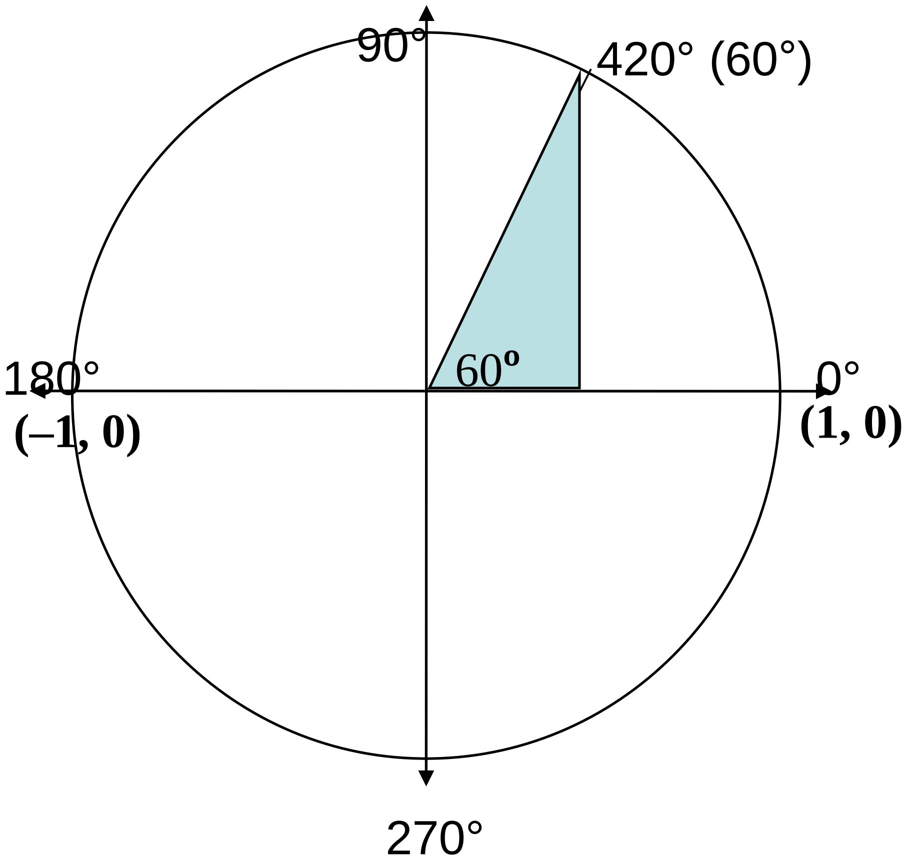
It is on an axis, so it is a special case where we need to use the fact that (x, y) = (cos θ, sin θ).

Sin 270° = -1

Cos 270° = 0

Tan 270° = -1/0 = *undefined*

Example: Find the sine, cosine, and tangent values of 420°.



Where is 420°?

420 is greater than 360, so subtract 420 – 360 = 60. 60 is greater than 0, less than 90, so it is in the 1st quadrant.

What is the reference angle?

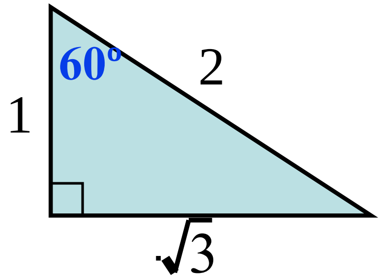
60°

Use our 30-60-90 triangle to find values:

Sin 60° = $\frac{\sqrt{3}}{2}$

Cos 60° = $\frac{1}{2}$

Tan 60° = $\frac{\sqrt{3}}{1} = \sqrt{3}$



It is in the first quadrant, ("all") so all values are positive. So my final answers are

Sin 420° = $\frac{\sqrt{3}}{2}$ Cos 420° = $\frac{1}{2}$ Tan 420° = $\sqrt{3}$