Exercise Set 6.7

Practice Exercises

In Exercises 1–8, use the given vectors to find $\mathbf{v}\cdot\mathbf{w}$ *and* $\mathbf{v}\cdot\mathbf{v}$.

1. $\mathbf{v} = 3\mathbf{i} + \mathbf{j}, \quad \mathbf{w} = \mathbf{i} + 3\mathbf{j}$
2. $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j}, \quad \mathbf{w} = \mathbf{i} + 4\mathbf{j}$
3. $\mathbf{v} = 5\mathbf{i} - 4\mathbf{j}, \quad \mathbf{w} = -2\mathbf{i} - \mathbf{j}$
4. $\mathbf{v} = 7\mathbf{i} - 2\mathbf{j}, \quad \mathbf{w} = -3\mathbf{i} - \mathbf{j}$
5. $\mathbf{v} = -6\mathbf{i} - 5\mathbf{j}, \quad \mathbf{w} = -10\mathbf{i} - 8\mathbf{j}$
6. $\mathbf{v} = -8\mathbf{i} - 3\mathbf{j}, \quad \mathbf{w} = -10\mathbf{i} - 5\mathbf{j}$
7. $\mathbf{v} = 5\mathbf{i}, \quad \mathbf{w} = \mathbf{j}$
8. $\mathbf{v} = \mathbf{i}, \quad \mathbf{w} = -5\mathbf{j}$

In Exercises 9–16, let

$$\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \quad \mathbf{v} = 3\mathbf{i} + \mathbf{j}, \quad \text{and} \quad \mathbf{w} = \mathbf{i} + 4\mathbf{j}.$$

Find each specified scalar.

9. $\mathbf{u}\cdot(\mathbf{v} + \mathbf{w})$
10. $\mathbf{v}\cdot(\mathbf{u} + \mathbf{w})$
11. $\mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{w}$
12. $\mathbf{v}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{w}$
13. $(4\mathbf{u})\cdot\mathbf{v}$
14. $(5\mathbf{v})\cdot\mathbf{w}$
15. $4(\mathbf{u}\cdot\mathbf{v})$
16. $5(\mathbf{v}\cdot\mathbf{w})$

In Exercises 17–22, find the angle between $\mathbf{v}$ *and* $\mathbf{w}$. *Round to the nearest tenth of a degree.*

17. $\mathbf{v} = 2\mathbf{i} - \mathbf{j}, \quad \mathbf{w} = 3\mathbf{i} + 4\mathbf{j}$
18. $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}, \quad \mathbf{w} = 3\mathbf{i} + 6\mathbf{j}$
19. $\mathbf{v} = -3\mathbf{i} + 2\mathbf{j}, \quad \mathbf{w} = 4\mathbf{i} - \mathbf{j}$
20. $\mathbf{v} = \mathbf{i} + 2\mathbf{j}, \quad \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$
21. $\mathbf{v} = 6\mathbf{i}, \quad \mathbf{w} = 5\mathbf{i} + 4\mathbf{j}$
22. $\mathbf{v} = 3\mathbf{j}, \quad \mathbf{w} = 4\mathbf{i} + 5\mathbf{j}$

In Exercises 23–32, use the dot product to determine whether $\mathbf{v}$ *and* $\mathbf{w}$ *are orthogonal.*

23. $\mathbf{v} = \mathbf{i} + \mathbf{j}, \quad \mathbf{w} = \mathbf{i} - \mathbf{j}$
24. $\mathbf{v} = \mathbf{i} + \mathbf{j}, \quad \mathbf{w} = -\mathbf{i} + \mathbf{j}$
25. $\mathbf{v} = 2\mathbf{i} + 8\mathbf{j}, \quad \mathbf{w} = 4\mathbf{i} - \mathbf{j}$
26. $\mathbf{v} = 8\mathbf{i} - 4\mathbf{j}, \quad \mathbf{w} = -6\mathbf{i} - 12\mathbf{j}$
27. $\mathbf{v} = 2\mathbf{i} - 2\mathbf{j}, \quad \mathbf{w} = -\mathbf{i} + \mathbf{j}$
28. $\mathbf{v} = 5\mathbf{i} - 5\mathbf{j}, \quad \mathbf{w} = \mathbf{i} - \mathbf{j}$
29. $\mathbf{v} = 3\mathbf{i}, \quad \mathbf{w} = -4\mathbf{i}$
30. $\mathbf{v} = 5\mathbf{i}, \quad \mathbf{w} = -6\mathbf{i}$
31. $\mathbf{v} = 3\mathbf{i}, \quad \mathbf{w} = -4\mathbf{j}$
32. $\mathbf{v} = 5\mathbf{i}, \quad \mathbf{w} = -6\mathbf{j}$

In Exercises 33–38, find $\text{proj}_{\mathbf{w}}\mathbf{v}$. *Then decompose* $\mathbf{v}$ *into two vectors,* $\mathbf{v}_1$ *and* $\mathbf{v}_2$, *where* $\mathbf{v}_1$ *is parallel to* $\mathbf{w}$ *and* $\mathbf{v}_2$ *is orthogonal to* $\mathbf{w}$.

33. $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}, \quad \mathbf{w} = \mathbf{i} - \mathbf{j}$
34. $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}, \quad \mathbf{w} = 2\mathbf{i} + \mathbf{j}$
35. $\mathbf{v} = \mathbf{i} + 3\mathbf{j}, \quad \mathbf{w} = -2\mathbf{i} + 5\mathbf{j}$
36. $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j}, \quad \mathbf{w} = -3\mathbf{i} + 6\mathbf{j}$
37. $\mathbf{v} = \mathbf{i} + 2\mathbf{j}, \quad \mathbf{w} = 3\mathbf{i} + 6\mathbf{j}$
38. $\mathbf{v} = 2\mathbf{i} + \mathbf{j}, \quad \mathbf{w} = 6\mathbf{i} + 3\mathbf{j}$

Practice Plus

In Exercises 39–42, let

$$\mathbf{u} = -\mathbf{i} + \mathbf{j}, \quad \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}, \quad \text{and} \quad \mathbf{w} = -5\mathbf{j}.$$

Find each specified scalar or vector.

39. $5\mathbf{u}\cdot(3\mathbf{v} - 4\mathbf{w})$
40. $4\mathbf{u}\cdot(5\mathbf{v} - 3\mathbf{w})$
41. $\text{proj}_{\mathbf{u}}(\mathbf{v} + \mathbf{w})$
42. $\text{proj}_{\mathbf{u}}(\mathbf{v} - \mathbf{w})$

In Exercises 43–44, find the angle, in degrees, between $\mathbf{v}$ *and* $\mathbf{w}$.

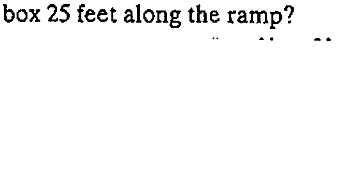
43. $\mathbf{v} = 2\cos\frac{4\pi}{3}\mathbf{i} + 2\sin\frac{4\pi}{3}\mathbf{j}, \quad \mathbf{w} = 3\cos\frac{3\pi}{2}\mathbf{i} + 3\sin\frac{3\pi}{2}\mathbf{j}$
44. $\mathbf{v} = 3\cos\frac{5\pi}{3}\mathbf{i} + 3\sin\frac{5\pi}{3}\mathbf{j}, \quad \mathbf{w} = 2\cos\pi\mathbf{i} + 2\sin\pi\mathbf{j}$

In Exercises 45–50, determine whether $\mathbf{v}$ *and* $\mathbf{w}$ *are parallel, orthogonal, or neither.*

45. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j}, \quad \mathbf{w} = 6\mathbf{i} - 10\mathbf{j}$
46. $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}, \quad \mathbf{w} = -6\mathbf{i} + 9\mathbf{j}$
47. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j}, \quad \mathbf{w} = 6\mathbf{i} + 10\mathbf{j}$
48. $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}, \quad \mathbf{w} = -6\mathbf{i} - 9\mathbf{j}$
49. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j}, \quad \mathbf{w} = 6\mathbf{i} + \frac{18}{5}\mathbf{j}$
50. $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}, \quad \mathbf{w} = -6\mathbf{i} - 4\mathbf{j}$

Application Exercises

51. The components of $\mathbf{v} = 240\mathbf{i} + 300\mathbf{j}$ represent the respective number of gallons of regular and premium gas sold at a station. The components of $\mathbf{w} = 2.90\mathbf{i} + 3.07\mathbf{j}$ represent the respective prices per gallon for each kind of gas. Find $\mathbf{v}\cdot\mathbf{w}$ and describe what the answer means in practical terms.
52. The components of $\mathbf{v} = 180\mathbf{i} + 450\mathbf{j}$ represent the respective number of one-day and three-day videos rented from a video store. The components of $\mathbf{w} = 3\mathbf{i} + 2\mathbf{j}$ represent the prices to rent the one-day and three-day videos, respectively. Find $\mathbf{v}\cdot\mathbf{w}$ and describe what the answer means in practical terms.
53. Find the work done in pushing a car along a level road from point A to point B, 80 feet from A, while exerting a constant force of 95 pounds. Round to the nearest foot-pound.
54. Find the work done when a crane lifts a 6000-pound boulder through a vertical distance of 12 feet. Round to the nearest foot-pound.
55. A wagon is pulled along level ground by exerting a force of 40 pounds on a handle that makes an angle of 32° with the horizontal. How much work is done pulling the wagon 100 feet? Round to the nearest foot-pound.
56. A wagon is pulled along level ground by exerting a force of 25 pounds on a handle that makes an angle of 38° with the horizontal. How much work is done pulling the wagon 100 feet? Round to the nearest foot-pound.
57. A force of 60 pounds on a rope is used to pull a box up a ramp inclined at 12° from the horizontal. The figure shows that the rope forms an angle of 38° with the horizontal. How much work is done pulling the box 20 feet along the ramp?



58. A force of 80 pounds on a rope is used to pull a box up a ramp inclined at 10° from the horizontal. The rope forms an angle of 33° with the horizontal. How much work is done pulling the box 25 feet along the ramp?