① $\cot\theta \csc\theta \tan^2\theta = \sec\theta$

$$\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{1}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right) = \sec\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$\sec\theta = \sec\theta$ ✓

② $\dfrac{\tan\theta}{\sec\theta} = \sin\theta$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \sin\theta$$

$\sin\theta = \sin\theta$ ✓

③ $\cos\theta = \sec\theta - \sin\theta\tan\theta$

$$\cos\theta = \frac{1}{\cos\theta} - \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$\cos\theta = \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$$

$$\cos\theta = \frac{1-\sin^2\theta}{\cos\theta}$$

$$\cos\theta = \frac{\cos^2\theta}{\cos\theta}$$

$\cos\theta = \cos\theta$ ✓

④ $\sin\theta + \cos\theta\cot\theta = \csc\theta$

$$\sin\theta + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right) = \csc\theta$$

$\sin\theta$: $\dfrac{\sin\theta}{1} + \dfrac{\cos^2\theta}{\sin\theta} = \csc\theta$

$$\frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta} = \csc\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \csc\theta$$

$$\frac{1}{\sin\theta} = \csc\theta$$

$\csc\theta = \csc\theta$ ✓

(5) $\dfrac{\sec\theta + \tan\theta}{\cos\theta + \cot\theta} = \sin\theta\sec^2\theta$

$$\frac{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\sin\theta}\cdot\frac{\cos\theta}{1} + \frac{\cos\theta}{\sin\theta}} = \sin\theta\sec^2\theta$$

$$\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\sin\theta\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}} = \sin\theta\sec^2\theta$$

$$\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\sin\theta\cos\theta+\cos\theta}{\sin\theta}} = \sin\theta\sec^2\theta$$

$$\frac{1+\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta\cos\theta+\cos\theta} = \sin\theta\sec^2\theta$$

$$\frac{1+\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta(\sin\theta+1)} = \sin\theta\sec^2\theta$$

$$\frac{\sin\theta}{\cos^2\theta} = \sin\theta\sec^2\theta$$

$$\sin\cdot\frac{1}{\cos^2\theta} = \sin\theta\sec^2\theta$$

$$\sin\theta\sec^2\theta = \sin\theta\sec^2\theta$$

✓

(6) $\dfrac{1+\sec\theta}{\tan\theta+\sin\theta} = \csc\theta$

$$\frac{\frac{\cos\theta}{\cos\theta}\cdot\frac{1}{1} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{1}\cdot\frac{\cos}{\cos}} = \csc\theta$$

$$\frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}} = \csc\theta$$

$$\frac{\cos\theta+1}{\sin\theta(1+\cos\theta)} = \csc\theta$$

$$\frac{1}{\sin\theta} = \csc\theta$$

$$\csc\theta = \csc\theta$$

✓

(7) $\dfrac{2\cos^2\theta - \sin^2\theta + 1}{\cos\theta} = 3\cos\theta$

$\dfrac{2\cos^2\theta - (1-\cos^2\theta) + 1}{\cos\theta} = 3\cos\theta$

$\dfrac{2\cos^2\theta - 1 + \cos^2\theta + 1}{\cos\theta} = 3\cos\theta$

$\dfrac{3\cos^2\theta}{\cos\theta} = 3\cos\theta$

$3\cos\theta = 3\cos\theta$ ✓

(8) $\dfrac{\cos\theta + \cot\theta}{\csc\theta + 1} = \cos\theta$

$\dfrac{\dfrac{\cos\theta}{1} + \dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta} + \dfrac{1}{1}} = \cos\theta$ (multiply by $\frac{\sin}{\sin}$)

$\dfrac{\dfrac{\sin\theta\cos\theta}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta} + \dfrac{\sin\theta}{\sin\theta}} = \cos\theta$

$\dfrac{\dfrac{\sin\theta\cos\theta + \cos\theta}{\sin\theta}}{\dfrac{1+\sin\theta}{\sin\theta}} = \cos\theta$

$\dfrac{\sin\theta\cos\theta + \cos\theta}{1+\sin\theta} = \cos\theta$

$\dfrac{\cos\theta(\sin\theta + 1)}{1+\sin\theta} = \cos\theta$

$\cos\theta = \cos\theta$ ✓

⑨ $\sin^2\theta\left(\frac{\csc^2\theta-1}{\cos^2\theta}-\cot^2\theta\right)=\sin^2\theta$

$\sin^2\theta\left(\frac{\cot^2\theta}{\cos^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}\right)=\sin^2\theta$

$\frac{\sin^2\theta\cot^2\theta}{\cos^2\theta}-\cos^2\theta=\sin^2\theta$

$\frac{\sin^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta}\right)}{\cos^2\theta}-\cos^2\theta=\sin^2\theta$

$\frac{\cos^2\theta}{\cos^2\theta}-\cos^2\theta=\sin^2\theta$

$1-\cos^2\theta=\sin^2\theta$

$\sin^2\theta=\sin^2\theta$ ✓

⑩ $(\sin\theta-\cos\theta)^2+2\sin\theta-\cos\theta=(1+2\sin\theta)(1-\cos\theta)$

$(\sin\theta-\cos\theta)(\sin\theta-\cos\theta)+2\sin\theta-\cos\theta=(1+2\sin\theta)(1-\cos\theta)$

$\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta+2\sin\theta-\cos\theta=(1+2\sin\theta)(1-\cos\theta)$

$1-2\sin\theta\cos\theta+2\sin\theta-\cos\theta=(1+2\sin\theta)(1-\cos\theta)$

$(2\sin\theta-2\sin\theta\cos\theta)+(1-\cos\theta)=(1+2\sin\theta)(1-\cos\theta)$

$2\sin\theta(1-\cos\theta)+1(1-\cos\theta)=(1+2\sin\theta)(1-\cos\theta)$

$(1+2\sin\theta)(1-\cos\theta)=(1+2\sin\theta)(1-\cos\theta)$ ✓